

STRONG GEODETIC PROBLEM IN NETWORKS

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Abstract

In order to model certain social network problems, the strong geodetic problem and its related invariant, the strong geodetic number, are introduced. The problem is conceptually similar to the classical geodetic problem but seems intrinsically more difficult. The strong geodetic number is compared with the geodetic number and with the isometric path number. It is determined for several families of graphs including Apollonian networks. Applying Sierpiński graphs, an algorithm is developed that returns a minimum path cover of Apollonian networks corresponding to the strong geodetic number. It is also proved that the strong geodetic problem is NP-complete.

Keywords: geodetic problem, strong geodetic problem, Apollonian networks, Sierpiński graphs, computational complexity.

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