

NICHE HYPERGRAPHS OF PRODUCTS OF DIGRAPHS

MARTIN SONNTAG

Faculty of Mathematics and Computer Science
TU Bergakademie Freiberg
Prüferstraße 1, D-09596 Freiberg, Germany
e-mail: sonntag@tu-freiberg.de

AND

HANNS-MARTIN TEICHERT

Institute of Mathematics
University of Lübeck
Ratzeburger Allee 160, D-23562 Lübeck, Germany
e-mail: teichert@math.uni-luebeck.de

Abstract

If $D = (V, A)$ is a digraph, its *niche hypergraph* $NH(D) = (V, \mathcal{E})$ has the edge set $\mathcal{E} = \{e \subseteq V \mid |e| \geq 2 \wedge \exists v \in V : e = N_D^-(v) \vee e = N_D^+(v)\}$. Niche hypergraphs generalize the well-known niche graphs and are closely related to competition hypergraphs as well as common enemy hypergraphs. For several products $D_1 \circ D_2$ of digraphs D_1 and D_2 , we investigate the relations between the niche hypergraphs of the factors D_1 , D_2 and the niche hypergraph of their product $D_1 \circ D_2$.

Keywords: niche hypergraph, product of digraphs, competition hypergraph.

2010 Mathematics Subject Classification: 05C65, 05C76, 05C20.

REFERENCES

- [1] J. Bang-Jensen and G. Gutin, *Digraphs: Theory, Algorithms and Applications* (Springer, London, 2001).
- [2] C. Cable, K.F. Jones, J.R. Lundgren and S. Seager, *Niche graphs*, *Discrete Appl. Math.* **23** (1989) 231–241.
doi:10.1016/0166-218X(89)90015-2

- [3] J.E. Cohen, Interval graphs and food webs: a finding and a problem, RAND Corp. Document 17696-PR (Santa Monica, CA, 1968).
- [4] M. Cozzens, *Food webs, competition graphs and habitat formation*, Math. Model. Nat. Phenom. **6** (2011) 22–38.
doi:10.1051/mmnp/20116602
- [5] C. Garske, M. Sonntag and H.-M. Teichert, *Niche hypergraphs*, Discuss. Math. Graph Theory **36** (2016) 819–832.
doi:10.7151/dmgt.1893
- [6] S.-R. Kim, *The competition number and its variants*, in: Quo Vadis, Graph Theory?, Ed(s) J. Gimbel, J.W. Kennedy and L.V. Quintas (Ann. Discrete Math. **55**, North Holland, Amsterdam, 1993).
doi:10.1016/S0167-5060(08)70396-0
- [7] S.-R. Kim, B. Park and Y. Sano, *The competition number of the complement of a cycle*, Discrete Appl. Math. **161** (2013) 1755–1760.
doi:10.1016/j.dam.2011.10.034
- [8] S.-R. Kim, B. Park and Y. Sano, *A generalization of Opsut’s result on the competition numbers of line graphs*, Discrete Appl. Math. **181** (2015) 152–159.
doi:10.1016/j.dam.2014.10.014
- [9] J. Kuhl, *Transversals and competition numbers of complete multipartite graphs*, Discrete Appl. Math. **161** (2013) 435–440.
doi:10.1016/j.dam.2012.09.012
- [10] B.-J. Li and G.J. Chang, *Competition numbers of complete r -partite graphs*, Discrete Appl. Math. **160** (2012) 2271–2276.
doi:10.1016/j.dam.2012.05.005
- [11] J.R. Lundgren, *Food webs, competition graphs, competition—common enemy graphs and niche graphs*, in: Applications of Combinatorics and Graph Theory to the Biological and Social Sciences, Ed(s) F. Roberts (IMA **17**, Springer, New York 1989) 221–243.
- [12] B.D. McKey, P. Schweitzer and P. Schweitzer, *Competition numbers, quasi line graphs and holes*, SIAM J. Discrete Math. **28** (2014) 77–91.
doi:10.1137/110856277
- [13] B. Park and Y. Sano, *On the hypercompetition numbers of hypergraphs*, Ars Combin. **100** (2011) 151–159.
- [14] B. Park and Y. Sano, *The competition numbers of ternary Hamming graphs*, Appl. Math. Lett. **24** (2011) 1608–1613.
doi:10.1016/j.aml.2011.04.012
- [15] B. Park and Y. Sano, *The competition number of a generalized line graph is at most two*, Discrete Math. Theor. Comput. Sci. **14** (2012) 1–10.
- [16] J. Park and Y. Sano, *The double competition hypergraph of a digraph*, Discrete Appl. Math. **195** (2015) 110–113.
doi:10.1016/j.dam.2014.04.001

- [17] Y. Sano, *On hypercompetition numbers of hypergraphs with maximum degree of at most two*, Discuss. Math. Graph Theory **35** (2015) 595–598.
doi:10.7151/dmgt.1826
- [18] D.D. Scott, *The competition-common enemy graph of a digraph*, Discrete Appl. Math. **17** (1987) 269–280.
doi:10.1016/0166-218X(87)90030-8
- [19] M. Sonntag and H.-M. Teichert, *Competition hypergraphs*, Discrete Appl. Math. **143** (2004) 324–329.
doi:10.1016/j.dam.2004.02.010
- [20] M. Sonntag and H.-M. Teichert, *Competition hypergraphs of products of digraphs*, Graphs Combin. **25** (2009) 611–624.
doi:10.1007/s00373-005-0868-9
- [21] M. Sonntag and H.-M. Teichert, *Products of digraphs and their competition graphs*, Discuss. Math. Graph Theory **36** (2016) 43–58.
doi:10.7151/dmgt.1851

Received 14 March 2017

Revised 13 March 2018

Accepted 13 March 2018