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TOTAL FORCING SETS AND ZERO FORCING SETS IN TREES

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Abstract

A dynamic coloring of the vertices of a graph G starts with an initial subset S of colored vertices, with all remaining vertices being non-colored. At each discrete time interval, a colored vertex with exactly one non-colored neighbor forces this non-colored neighbor to be colored. The initial set S is called a forcing set of G if, by iteratively applying the forcing process, every vertex in G becomes colored. If the initial set S has the added property that it induces a subgraph of G without isolated vertices, then S is called a total forcing set in G. The minimum cardinality of a total forcing set in G is its total forcing number, denoted $F_t(G)$. We prove that if T is a tree of order $n \geq 3$ with maximum degree Δ and with n_1 leaves, then $n_1 \leq F_t(T) \leq \frac{1}{\Delta}((\Delta - 1)n + 1)$. In both lower and upper bounds, we characterize the infinite family of trees achieving equality. Further we show that $F_t(T) \geq F(T) + 1$, and we characterize the extremal trees for which equality holds.

Keywords: forcing set, forcing number, total forcing set, total forcing number.

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