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THE SLATER AND SUB-k-DOMINATION NUMBER OF A GRAPH WITH APPLICATIONS TO DOMINATION AND k-DOMINATION

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Abstract

In this paper we introduce and study a new graph invariant derived from the degree sequence of a graph G, called the *sub-k-domination number* and denoted $\mathrm{sub}_k(G)$. This invariant serves as a generalization of the *Slater number*; in particular, we show that $\mathrm{sub}_k(G)$ is a computationally efficient sharp lower bound on the k-domination number of G, and improves on several known lower bounds. We also characterize the sub-k-domination numbers of several families of graphs, provide structural results on sub-k-domination, and explore properties of graphs which are $\mathrm{sub}_k(G)$-critical with respect to addition and deletion of vertices and edges.

Keywords: Slater number, domination number, sub-k-domination number, k-domination number, degree sequence index strategy.

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