

THE SLATER AND SUB- k -DOMINATION NUMBER OF A GRAPH WITH APPLICATIONS TO DOMINATION AND k -DOMINATION

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Abstract

In this paper we introduce and study a new graph invariant derived from the degree sequence of a graph G , called the *sub- k -domination number* and denoted $\text{sub}_k(G)$. This invariant serves as a generalization of the *Slater number*; in particular, we show that $\text{sub}_k(G)$ is a computationally efficient sharp lower bound on the k -domination number of G , and improves on several known lower bounds. We also characterize the sub- k -domination numbers of several families of graphs, provide structural results on sub- k -domination, and explore properties of graphs which are $\text{sub}_k(G)$ -critical with respect to addition and deletion of vertices and edges.

Keywords: Slater number, domination number, sub- k -domination number, k -domination number, degree sequence index strategy.

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