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LONGER CYCLES IN ESSENTIALLY 4-CONNECTED PLANAR GRAPHS

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Abstract

A planar 3-connected graph G is called *essentially* 4-*connected* if, for every 3-separator S, at least one of the two components of $G - S$ is an isolated vertex. Jackson and Wormald proved that the length $\mathrm{circ}(G)$ of a longest cycle of any essentially 4-connected planar graph G on n vertices is at least $\frac{2n+4}{5}$ and Fabrici, Harant and Jendroľ improved this result to $\mathrm{circ}(G) \geq \frac{1}{2}(n+4)$. In the present paper, we prove that an essentially 4-connected planar graph on n vertices contains a cycle of length at least $\frac{3}{5}(n+2)$ and that such a cycle can be found in time $O(n^2)$.

Keywords: essentially 4-connected planar graph, longest cycle, circumference, shortness coefficient.

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