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DECOMPOSITIONS OF CUBIC TRACEABLE GRAPHS

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Abstract

A *traceable graph* is a graph with a Hamilton path. The 3-Decomposition Conjecture states that every connected cubic graph can be decomposed into a spanning tree, a 2-regular graph and a matching. We prove the conjecture for cubic traceable graphs.

Keywords: decomposition, cubic traceable graph, spanning tree, matching, 2-regular graph.

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