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MORE ON THE MINIMUM SIZE OF GRAPHS WITH GIVEN RAINBOW INDEX

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Abstract

The concept of k-rainbow index $rx_k(G)$ of a connected graph G, introduced by Chartrand *et al.*, is a natural generalization of the rainbow connection number of a graph. Liu introduced a parameter $t(n, k, \ell)$ to investigate the problems of the minimum size of a connected graph with given order and k-rainbow index at most ℓ and obtained some exact values and upper bounds for $t(n, k, \ell)$. In this paper, we obtain some exact values of $t(n, k, \ell)$ for large ℓ and better upper bounds of $t(n, k, \ell)$ for small ℓ and $k = 3$.

Keywords: Steiner distance, rainbow S-tree, k-rainbow index.

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