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AN IMPROVED UPPER BOUND ON NEIGHBOR EXPANDED SUM DISTINGUISHING INDEX

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Abstract

A total k-weighting f of a graph G is an assignment of integers from the set $\{1,\dots,k\}$ to the vertices and edges of G. We say that f is neighbor expanded sum distinguishing, or NESD for short, if $\sum_{w\in N(v)}\big(f(vw)+f(w)\big)$ differs from $\sum_{w\in N(u)}(f(uw)+f(w))$ for every two adjacent vertices v and u of G. The neighbor expanded sum distinguishing index of G, denoted by $\mathrm{egndi}_{\sum}(G)$, is the minimum positive integer k for which there exists an NESD weighting of G. An NESD weighting was introduced and investigated by Flandrin *et al.* (2017), where they conjectured that $\mathrm{egndi}_{\sum}(G)\le 2$ for any graph G. They examined some special classes of graphs, while proving that $\mathrm{egndi}_{\sum}(G)\le\chi(G)+1$. We improve this bound and show that $\mathrm{egndi}_{\sum}(G)\le 3$ for any graph G. We also show that the conjecture holds for all bipartite, 3-regular and 4-regular graphs.

Keywords: general edge coloring, total coloring, neighbor sum distinguishing index.

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References

[1] G. Chartrand and P. Zhang, Chromatic Graph Theory (Boca Raton: Chapman & Hall/CRC, 2009).

[2] E. Flandrin, H. Li, A. Marczyk, J.-F. Sacle and M. Woźniak, *A note on neighbor expanded sum distinguishing index*, Discuss. Math. Graph Theory **37** (2017) 29–37. doi:10.7151/dmgt.1909

[3] M. Karoński, T. Łuczak and A. Thomason, *Edge weights and vertex colours*, J. Combin. Theory Ser. B **91** (2004) 151–157. doi:10.1016/j.jctb.2003.12.001

[4] M. Kalkowski, M. Karoński and F. Pfender, *Vertex-coloring edge-weightings: Towards the* 1-2-3-*conjecture*, J. Combin. Theory Ser. B **100** (2010) 347–349. doi:10.1016/j.jctb.2009.06.002

[5] R.L. Brooks, *On coloring the nodes of a network*, Proc. Cambridge Philos. Soc. **37** (1941) 194–197. doi:10.1017/S030500410002168X

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