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AN IMPROVED UPPER BOUND ON NEIGHBOR EXPANDED SUM DISTINGUISHING INDEX

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Abstract

A total k-weighting f of a graph G is an assignment of integers from the set $\{1, \dots, k\}$ to the vertices and edges of G. We say that f is neighbor expanded sum distinguishing, or NESD for short, if $\sum_{w \in N(v)} \big(f(vw) + f(w)\big)$ differs from $\sum_{w \in N(u)}(f(uw) + f(w))$ for every two adjacent vertices v and u of G. The neighbor expanded sum distinguishing index of G, denoted by $\text{egndi}_{\sum}(G)$, is the minimum positive integer k for which there exists an NESD weighting of G. An NESD weighting was introduced and investigated by Flandrin *et al.* (2017), where they conjectured that $\text{egndi}_{\sum}(G) \le 2$ for any graph G. They examined some special classes of graphs, while proving that $\text{egndi}_{\sum}(G) \le \chi(G) + 1$. We improve this bound and show that $\text{egndi}_{\sum}(G) \le 3$ for any graph G. We also show that the conjecture holds for all bipartite, 3-regular and 4-regular graphs.

Keywords: general edge coloring, total coloring, neighbor sum distinguishing index.

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