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THE COMPARED COSTS OF DOMINATION LOCATION-DOMINATION AND IDENTIFICATION

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Abstract

Let $G = (V, E)$ be a finite graph and $r \geq 1$ be an integer. For $v \in V$, let $B_r(v) = \{x \in V : d(v, x) \leq r\}$ be the ball of radius r centered at v. A set $C \subseteq V$ is an r-dominating code if for all $v \in V$, we have $B_r(v) \cap C \neq \emptyset$; it is an r-locating-dominating code if for all $v \in V$, we have $B_r(v) \cap C \neq \emptyset$, and for any two distinct non-codewords $x \in V \setminus C$, $y \in V \setminus C$, we have $B_r(x) \cap C \neq B_r(y) \cap C$; it is an r-identifying code if for all $v \in V$, we have $B_r(v) \cap C \neq \emptyset$, and for any two distinct vertices $x \in V$, $y \in V$, we have $B_r(x) \cap C \neq B_r(y) \cap C$. We denote by $\gamma_r(G)$ (respectively, $ld_r(G)$ and $id_r(G)$) the smallest possible cardinality of an r-dominating code (respectively, an r-locating-dominating code and an r-identifying code). We study how small and how large the three differences $id_r(G) - ld_r(G)$, $id_r(G) - \gamma_r(G)$ and $ld_r(G) - \gamma_r(G)$ can be.

Keywords: graph theory, dominating set, locating-dominating code, identifying code, twin-free graph.

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