

THE COMPARED COSTS OF DOMINATION LOCATION-DOMINATION AND IDENTIFICATION

OLIVIER HUDRY

*LTCI, Télécom ParisTech
Université Paris-Saclay
46 rue Barrault, 75634 Paris Cedex 13 – France
e-mail: olivier.hudry@telecom-paristech.fr*

AND

ANTOINE LOBSTEIN

*Centre National de la Recherche Scientifique
Laboratoire de Recherche en Informatique, UMR 8623
Université Paris-Sud, Université Paris-Saclay
Bâtiment 650 Ada Lovelace, 91405 Orsay Cedex – France
e-mail: antoine.lobstein@lri.fr*

Abstract

Let $G = (V, E)$ be a finite graph and $r \geq 1$ be an integer. For $v \in V$, let $B_r(v) = \{x \in V : d(v, x) \leq r\}$ be the ball of radius r centered at v . A set $C \subseteq V$ is an r -dominating code if for all $v \in V$, we have $B_r(v) \cap C \neq \emptyset$; it is an r -locating-dominating code if for all $v \in V$, we have $B_r(v) \cap C \neq \emptyset$, and for any two distinct non-codewords $x \in V \setminus C$, $y \in V \setminus C$, we have $B_r(x) \cap C \neq B_r(y) \cap C$; it is an r -identifying code if for all $v \in V$, we have $B_r(v) \cap C \neq \emptyset$, and for any two distinct vertices $x \in V$, $y \in V$, we have $B_r(x) \cap C \neq B_r(y) \cap C$. We denote by $\gamma_r(G)$ (respectively, $ld_r(G)$ and $id_r(G)$) the smallest possible cardinality of an r -dominating code (respectively, an r -locating-dominating code and an r -identifying code). We study how small and how large the three differences $id_r(G) - ld_r(G)$, $id_r(G) - \gamma_r(G)$ and $ld_r(G) - \gamma_r(G)$ can be.

Keywords: graph theory, dominating set, locating-dominating code, identifying code, twin-free graph.

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REFERENCES

- [1] C. Berge, *Graphes* (Gauthier-Villars, Paris, 1983).
- [2] C. Berge, *Graphs* (North-Holland Publishing Co., Amsterdam, 1985).
- [3] N. Bertrand, *Codes identifiants et codes localisateurs-dominateurs sur certains graphes*, Mémoire de stage de maîtrise, ENST (Paris, France, 2001).
- [4] I. Charon, O. Hudry and A. Lobstein, *Possible cardinalities for identifying codes in graphs*, Australas. J. Combin. **32** (2005) 177–195.
- [5] I. Charon, O. Hudry and A. Lobstein, *Possible cardinalities for locating-dominating codes in graphs*, Australas. J. Combin. **34** (2006) 23–32.
- [6] I. Charon, O. Hudry and A. Lobstein, *Extremal cardinalities for identifying and locating-dominating codes in graphs*, Discrete Math. **307** (2007) 356–366.
doi:10.1016/j.disc.2005.09.027
- [7] C.J. Colbourn, P.J. Slater and L.K. Stewart, *Locating dominating sets in series parallel networks*, Congr. Numer. **56** (1987) 135–162.
- [8] R. Diestel, *Graph Theory* (Springer-Verlag, Berlin, 2005).
- [9] J.F. Fink, M.S. Jacobson, L.F. Kinch and J. Roberts, *On graphs having domination number half their order*, Period. Math. Hungar. **16** (1985) 287–293.
doi:10.1007/BF01848079
- [10] F. Foucaud, E. Guerrini, M. Kovše, R. Naserasr, A. Parreau and P. Valicov, *Extremal graphs for the identifying code problem*, European J. Combin. **32** (2011) 628–638.
doi:10.1016/j.ejc.2011.01.002
- [11] S. Gravier, R. Klasing and J. Moncel, *Hardness results and approximation algorithms for identifying codes and locating-dominating codes in graphs*, Algorithmic Oper. Res. **3** (2008) 43–50.
- [12] S. Gravier and J. Moncel, *On graphs having a $V \setminus \{x\}$ set as an identifying code*, Discrete Math. **307** (2007) 432–434.
doi:10.1016/j.disc.2005.09.035
- [13] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs* (Marcel Dekker, New York, 1998).
- [14] M.G. Karpovsky, K. Chakrabarty and L.B. Levitin, *On a new class of codes for identifying vertices in graphs*, IEEE Trans. Inform. Theory **44** (1998) 599–611.
doi:10.1109/18.661507
- [15] A. Lobstein, *Watching systems, identifying, locating-dominating and discriminating codes in graphs, a bibliography*.
<https://www.lri.fr/~lobstein/debutBIBidetlocdom.pdf>
- [16] O. Ore, *Theory of Graphs* (American Mathematical Society, Providence, 1962).
doi:10.1090/coll/038

- [17] C. Payan and N.H. Xuong, *Domination-balanced graphs*, J. Graph Theory **6** (1982) 23–32.
doi:10.1002/jgt.3190060104
- [18] P.J. Slater, Domination and location in graphs, Research Report 93 (National University of Singapore, 1983).

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