

2-CONNECTED HAMILTONIAN CLAW-FREE GRAPHS INVOLVING DEGREE SUM OF ADJACENT VERTICES

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Abstract

For a graph H , define $\bar{\sigma}_2(H) = \min\{d(u) + d(v) \mid uv \in E(H)\}$. Let H be a 2-connected claw-free simple graph of order n with $\delta(H) \geq 3$. In [J. Graph Theory 86 (2017) 193–212], Chen proved that if $\bar{\sigma}_2(H) \geq \frac{n}{2} - 1$ and n is sufficiently large, then H is Hamiltonian with two families of exceptions. In this paper, we refine the result. We focus on the condition $\bar{\sigma}_2(H) \geq \frac{2n}{5} - 1$, and characterize non-Hamiltonian 2-connected claw-free graphs H of order n sufficiently large with $\bar{\sigma}_2(H) \geq \frac{2n}{5} - 1$. As byproducts, we prove that there are exactly six graphs in the family of 2-edge-connected triangle-free graphs of order at most seven that have no spanning closed trail and give an improvement of a result of Veldman in [Discrete Math. 124 (1994) 229–239].

Keywords: Hamiltonian cycle, degree sum, dominating closed trail, closure.

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