Discussiones Mathematicae
Graph Theory 40 (2020) 85–106
doi:10.7151/dmgt.2125

2-CONNECTED HAMILTONIAN CLAW-FREE GRAPHS INVOLVING DEGREE SUM OF ADJACENT VERTICES

Tao Tian

School of Mathematics and Statistics
Beijing Institute of Technology
Beijing 102488, P.R. China

Faculty of EEMCS, University of Twente
P.O. Box 217, 7500 AE Enschede, The Netherlands

e-mail: taotian0118@163.com

AND

Liming Xiong

School of Mathematics and Statistics
Beijing Key Laboratory on MCAACI
Beijing Institute of Technology
Beijing 102488, P.R. China

e-mail: lmxiong@bit.edu.cn

Abstract

For a graph H, define $\bar{\sigma}_2(H) = \min\{d(u) + d(v)|\ uv \in E(H)\}$. Let H be a 2-connected claw-free simple graph of order n with $\delta(H) \geq 3$. In [J. Graph Theory 86 (2017) 193–212], Chen proved that if $\bar{\sigma}_2(H) \geq \frac{n}{2} - 1$ and n is sufficiently large, then H is Hamiltonian with two families of exceptions. In this paper, we refine the result. We focus on the condition $\bar{\sigma}_2(H) \geq \frac{2n}{5} - 1$, and characterize non-Hamiltonian 2-connected claw-free graphs H of order n sufficiently large with $\bar{\sigma}_2(H) \geq \frac{2n}{5} - 1$. As byproducts, we prove that there are exactly six graphs in the family of 2-edge-connected triangle-free graphs of order at most seven that have no spanning closed trail and give an improvement of a result of Veldman in [Discrete Math. 124 (1994) 229–239].

Keywords: Hamiltonian cycle, degree sum, dominating closed trail, closure.

2010 Mathematics Subject Classification: 05C45.

References

[1] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, 2008).

[2] P.A. Catlin, *A reduction method to find spanning Eulerian subgraphs*, J. Graph Theory **12** (1988) 29–45.
doi:10.1016/S0012-365X(95)00149-Q

[3] P.A. Catlin, Z. Han and H.-J Lai, *Graphs without spanning Eulerian trails*, Discrete Math. **160** (1996) 81–91.
doi:10.1002/jgt.22120

[4] Z.-H. Chen, *Hamiltonicity and degrees of adjacent vertices in claw-free graphs*, J. Graph Theory **86** (2017) 193–212.
doi:10.1002/jgt.22120

[5] G.H. Fan, *New sufficient conditions for cycles in graphs*, J. Combin. Theory Ser. B **37** (1981) 221–227.
doi:10.1016/0095-8956(84)90054-6

[6] R. Faudree, E. Flandrin and Z. Ryjáček, *Claw-free graphs—A survey*, Discrete Math. **164** (1997) 87–147.
doi:10.1016/S0012-365X(96)00045-3

[7] O. Favaron, E. Flandrin, H. Li and Z. Ryjáček, *Clique covering and degree conditions for Hamiltonicity in claw-free graphs*, Discrete Math. **236** (2001) 65–80.
doi:10.1016/S0012-365X(00)00432-5

[8] R. Gould, *Recent advances on the Hamiltonian problem: Survey* III, Graphs Combin. **30** (2014) 1–46.
doi:10.1007/s00373-013-1377-x

[9] F. Harary and C.St.J.A. Nash-Williams, *On Eulerian and Hamiltonian graphs and line graphs*, Canad. Math. Bull. **8** (1965) 701–710.
doi:10.4153/CMB-1965-051-3

[10] O. Kovářík, M. Mulač and Z. Ryjáček, *A note on degree conditions for Hamiltonicity in* 2*-connected claw-free graphs*, Discrete Math. **244** (2002) 253–268.
doi:10.1016/S0012-365X(01)00088-7

[11] H. Li, M. Lu, F. Tian and B. Wei, *Hamiltonian cycles in* 2*-connected claw-free graphs with at most* $5\delta - 5$ *vertices*, Congr. Numer. **122** (1996) 184–202.

[12] M.M. Matthews and D.P. Sumner, *Longest path and cycle in* $K_{1,3}$*-free graphs*, J. Graph Theory **9** (1985) 269–277.
doi:10.1002/jgt.3190090208

[13] O. Ore, *Note on Hamilton circuits*, Amer. Math. Monthly **67** (1960) 55.
doi:10.2307/2308928

[14] Z. Ryjáček, *On a closure concept in claw-free graphs*, J. Combin. Theory Ser. B **70** (1997) 217–224.
doi:10.1006/jctb.1996.1732

[15] Y. Shao, Claw-free graphs and line graphs, Ph.D dissertation (West Virginia University, 2005).

[16] H.J. Veldman, *On dominating and spanning circuits in graphs*, Discrete Math. **124** (1994) 229–239.
doi:10.1016/0012-365X(92)00063-W

[17] C.Q. Zhang, *Hamilton cycles in claw-free graphs*, J. Graph Theory **12** (1988) 209–216.
doi:10.1002/jgt.3190120211

Received 19 May 2017
Revised 6 February 2018
Accepted 6 February 2018