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A NOTE ON CYCLES IN LOCALLY HAMILTONIAN AND LOCALLY HAMILTON-CONNECTED GRAPHS

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Abstract

Let $\mathcal{P}$ be a property of a graph. A graph G is said to be locally $\mathcal{P}$, if the subgraph induced by the open neighbourhood of every vertex in G has property $\mathcal{P}$. Ryjáček conjectures that every connected, locally connected graph is weakly pancyclic. Motivated by the above conjecture, van Aardt *et al.* [S.A.van Aardt, M. Frick, O.R. Oellermann and J.P.de Wet, *Global cycle properties in locally connected, locally traceable and locally Hamiltonian graphs*, Discrete Appl. Math. 205 (2016) 171–179] investigated the global cycle structures in connected, locally traceable/Hamiltonian graphs. Among other results, they proved that a connected, locally Hamiltonian graph G with maximum degree at least $|V(G)| - 5$ is weakly pancyclic. In this note, we improve this result by showing that such a graph with maximum degree at least $|V(G)|-6$ is weakly pancyclic. Furthermore, we show that a connected, locally Hamilton-connected graph with maximum degree at most 7 is fully cycle extendable.

Keywords: locally connected, locally Hamiltonian, locally Hamilton-connected, fully cycle extendability, weakly pancyclicity.

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