

## NEIGHBOR SUM DISTINGUISHING TOTAL CHROMATIC NUMBER OF PLANAR GRAPHS WITHOUT 5-CYCLES<sup>1</sup>

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### Abstract

For a given graph  $G = (V(G), E(G))$ , a proper total coloring  $\phi : V(G) \cup E(G) \rightarrow \{1, 2, \dots, k\}$  is neighbor sum distinguishing if  $f(u) \neq f(v)$  for each edge  $uv \in E(G)$ , where  $f(v) = \sum_{uv \in E(G)} \phi(uv) + \phi(v)$ ,  $v \in V(G)$ . The smallest integer  $k$  in such a coloring of  $G$  is the neighbor sum distinguishing total chromatic number, denoted by  $\chi''_\Sigma(G)$ . Pilśniak and Woźniak first introduced this coloring and conjectured that  $\chi''_\Sigma(G) \leq \Delta(G) + 3$  for any graph with maximum degree  $\Delta(G)$ . In this paper, by using the discharging method, we prove that for any planar graph  $G$  without 5-cycles,  $\chi''_\Sigma(G) \leq \max\{\Delta(G) + 2, 10\}$ . The bound  $\Delta(G) + 2$  is sharp. Furthermore, we get the exact value of  $\chi''_\Sigma(G)$  if  $\Delta(G) \geq 9$ .

**Keywords:** neighbor sum distinguishing total coloring, discharging method, planar graph.

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