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DEFICIENCY AND FORBIDDEN SUBGRAPHS OF CONNECTED, LOCALLY-CONNECTED GRAPHS[1]

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Abstract

A graph G is *locally-connected* if the neighbourhood $N_G(v)$ induces a connected subgraph for each vertex v in G. For a graph G, the *deficiency* of G is the number of vertices unsaturated by a maximum matching, denoted by $\operatorname{def}(G)$. In fact, the deficiency of a graph measures how far a maximum matching is from being perfect matching. Saito and Xiong have studied subgraphs, the absence of which forces a connected and locally-connected graph G of sufficiently large order to satisfy $\operatorname{def}(G) \leq 1$. In this paper, we extend this result to the condition of $\operatorname{def}(G) \leq k$, where k is a positive integer. Let $\beta_0 = \lceil \frac{1}{2}(3+\sqrt{8k+17}) \rceil - 1$, we show that $K_{1,2}, K_{1,3}, \ldots, K_{1,\beta_0}$, K_3 or $K_2 \vee 2K_1$ is the required forbidden subgraph. Furthermore, we obtain some similar results about 3-connected, locally-connected graphs.

Keywords: deficiency, locally-connected graph, matching, forbidden subgraph.

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