*Discussiones Mathematicae*
*Graph Theory 40 (2020) 181–194*
*doi:10.7151/dmgt.2118*



# ON TOTAL $H$-IRREGULARITY STRENGTH OF THE DISJOINT UNION OF GRAPHS

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**Abstract**

A simple graph $G$ admits an $H$-covering if every edge in $E(G)$ belongs to at least to one subgraph of $G$ isomorphic to a given graph $H$. For the subgraph $H \subseteq G$ under a total $k$-labeling we define the associated $H$-weight as the sum of labels of all vertices and edges belonging to $H$. The total $k$-labeling is called the *$H$-irregular total $k$-labeling* of a graph $G$ admitting

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an $H$-covering if all subgraphs of $G$ isomorphic to $H$ have distinct weights. The *total $H$-irregularity strength* of a graph $G$ is the smallest integer $k$ such that $G$ has an $H$-irregular total $k$-labeling.

In this paper, we estimate lower and upper bounds on the total $H$-irregularity strength for the disjoint union of multiple copies of a graph and the disjoint union of two non-isomorphic graphs. We also prove the sharpness of the upper bounds.

**Keywords:** $H$-covering, $H$-irregular labeling, total $H$-irregularity strength, copies of graphs, union of graphs.

**2010 Mathematics Subject Classification:** 05C78, 05C70.

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Received 12 April 2017
Revised 5 February 2018
Accepted 22 February 2018