

ON TOTAL H -IRREGULARITY STRENGTH OF THE DISJOINT UNION OF GRAPHS

FARAHA ASHRAF

*Abdus Salam School of Mathematical Sciences
GC University, Lahore, Pakistan*

e-mail: faraha27@gmail.com

SUSANA CLARA LÓPEZ

*Dept. Matemàtiques, Universitat Politècnica de Catalunya
C/Esteve Terrades 5, 08860 Castelldefels, Spain*

e-mail: susana.clara.lopez@upc.edu

FRANCESC ANTONI MUNTANER-BATLE, AKITO OSHIMA

*Graph Theory and Applications Research Group, School of Electrical Engineering
and Computer Science, Faculty of Engineering and Built Environment
The University of Newcastle, Newcastle, Australia*

e-mail: fambles@yahoo.es
akitoism@yahoo.co.jp

AND

MARTIN BAČA¹, ANDREA SEMANIČOVÁ-FEŇOVČÍKOVÁ

*Department of Applied Mathematics and Informatics
Technical University, Košice, Slovakia*

e-mail: martin.baca@tuke.sk
andrea.fenovcikova@tuke.sk

Abstract

A simple graph G admits an H -covering if every edge in $E(G)$ belongs to at least to one subgraph of G isomorphic to a given graph H . For the subgraph $H \subseteq G$ under a total k -labeling we define the associated H -weight as the sum of labels of all vertices and edges belonging to H . The total k -labeling is called the H -irregular total k -labeling of a graph G admitting

¹Corresponding author.

an H -covering if all subgraphs of G isomorphic to H have distinct weights. The *total H -irregularity strength* of a graph G is the smallest integer k such that G has an H -irregular total k -labeling.

In this paper, we estimate lower and upper bounds on the total H -irregularity strength for the disjoint union of multiple copies of a graph and the disjoint union of two non-isomorphic graphs. We also prove the sharpness of the upper bounds.

Keywords: H -covering, H -irregular labeling, total H -irregularity strength, copies of graphs, union of graphs.

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