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CONFLICT-FREE VERTEX-CONNECTIONS OF GRAPHS

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Abstract

A path in a vertex-colored graph is called *conflict-free* if there is a color used on exactly one of its vertices. A vertex-colored graph is said to be *conflict-free vertex-connected* if any two vertices of the graph are connected by a conflict-free path. This paper investigates the question: for a connected graph G, what is the smallest number of colors needed in a vertex-coloring of G in order to make G conflict-free vertex-connected. As a result, we get that the answer is easy for 2-connected graphs, and very difficult for connected graphs with more cut-vertices, including trees.

Keywords: vertex-coloring, conflict-free vertex-connection, 2-connected graph, tree.

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