THE PROPER DIAMETER OF A GRAPH

Vincent Coll

Lehigh University, Department of Mathematics
Bethlehem, PA 18015, USA

e-mail: vec208@lehigh.edu

Jonelle Hook

Mount St. Mary's University
Department of Mathematics and Computer Science
Emmitsburg, MD 21727, USA

e-mail: jhook@msmary.edu

Colton Magnant

Georgia Southern University
Department of Mathematical Sciences
Statesboro, GA 30460, USA

e-mail: cmagnant@georgiasouthern.edu

Karen McCready

King's College
Department of Mathematics and Computer Science
Wilkes-Barre, PA 18711, USA

e-mail: karenmccready@kings.edu

AND

Kathleen Ryan

DeSales University
Department of Mathematics and Computer Science
Center Valley, PA 18034, USA

e-mail: kathleen.ryan@desales.edu

Abstract

A proper edge-coloring of a graph is a coloring in which adjacent edges receive distinct colors. A path is properly colored if consecutive edges have distinct colors, and an edge-colored graph is properly connected if there exists a properly colored path between every pair of vertices. In such a graph, we introduce the notion of the graph's proper diameter—which is a function of both the graph and the coloring— and define it to be the maximum length of a shortest properly colored path between any two vertices in the graph. We consider various families of graphs to find bounds on the gap between the diameter and possible proper diameters, paying singular attention to 2-colorings.

Keywords: diameter, properly connected, proper diameter.

2010 Mathematics Subject Classification: 05C12, 05C15.

References

[1] E. Andrews, E. Laforge, C. Lumduanhom and P. Zhang, *On proper-path colorings in graphs*, J. Combin. Math. Combin. Comput. **97** (2016) 189–207.

[2] V. Borozan, S. Fujita, A. Gerek, C. Magnant, Y. Manoussakis, L. Montero and Zs. Tuza, *Proper connection of graphs*, Discrete Math. **312** (2012) 2550–2560.
doi:10.1016/j.disc.2011.09.003

[3] G. Chartrand, G. L. Johns, A. K. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.

[4] G. Chartrand, G. Johns, K. McKeon and P. Zhang, *The rainbow connectivity of a graph*, Networks **54** (2009) 75–81.
doi:10.1002/net.20296

[5] S. Fujita, A. Gerek and C. Magnant, *Proper connection with many colors*, J. Comb. **3** (2012) 683–693.
doi:10.4310/JOC.2012.v3.n4.a6

[6] E. Laforge, C. Lumduanhom and P. Zhang, *Characterizations of graphs having large proper connection numbers*, Discuss. Math. Graph Theory **36** (2016) 439–453.
doi:10.7151/dmgt.1867

[7] E. Laforge, C. Lumduanhom and P. Zhang, *Chromatic-connection in graphs*, Congr. Numer. **225** (2015) 37–54.

[8] X. Li and C. Magnant, *Properly colored notions of connectivity—a dynamic survey*, Theory Appl. Graphs **0** (2015), Iss. 1, Article 2.
doi:10.20429/tag.2015.000102