

ON THE INDEPENDENCE NUMBER OF TRACEABLE 2-CONNECTED CLAW-FREE GRAPHS

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Abstract

A well-known theorem by Chvátal-Erdős [A note on Hamilton circuits, Discrete Math. 2 (1972) 111–135] states that if the independence number of a graph G is at most its connectivity plus one, then G is traceable. In this article, we show that every 2-connected claw-free graph with independence number $\alpha(G) \leq 6$ is traceable or belongs to two exceptional families of well-defined graphs. As a corollary, we also show that every 2-connected claw-free graph with independence number $\alpha(G) \leq 5$ is traceable.

Keywords: traceability, independence number, matching number, trail, closure.

2010 Mathematics Subject Classification: 05C38, 05C69, 05C70.

REFERENCES

- [1] J.A. Bondy and U.S.R. Murty, Graph Theory, Graduate in Mathematics (Springer, 2008).
- [2] S. Brandt, O. Favaron and Z. Ryjáček, *Closure and stable Hamiltonian properties in claw-free graphs*, J. Graph Theory **34** (2000) 30–41.
doi:10.1002/(SICI)1097-0118(200005)34:1<30::AID-JGT4>3.0.CO;2-R
- [3] Z. Chen, *Chvátal-Erdős type conditions for Hamiltonicity of claw-free graphs*, Graphs Combin. **32** (2016) 2253–2266.
doi:10.1007/s00373-016-1716-9

- [4] V. Chvátal and P. Erdős, *A note on Hamilton circuits*, Discrete Math. **2** (1972) 111–113.
doi:10.1016/0012-365X(72)90079-9
- [5] R. Faudree, E. Flandrin and Z. Ryjáček, *Claw-free graphs—A survey*, Discrete Math. **164** (1997) 87–147.
doi:10.1016/S0012-365X(96)00045-3
- [6] E. Flandrin and H. Li, *Further result on neighbourhood intersections*, Report de Recherche, L.R.I. **416**, Univ. Paris-Sud, Orsay (1989).
- [7] F. Harary and C. Nash-Williams, *On Eulerian and Hamiltonian graphs and line graphs*, Canad. Math. Bull. **8** (1965) 701–710.
doi:10.4153/CMB-1965-051-3
- [8] D. Li, H. Lai and M. Zhan, *Eulerian subgraphs and Hamilton-connected line graphs*, Discrete Appl. Math. **145** (2005) 422–428.
doi:10.1016/j.dam.2004.04.005
- [9] Z. Niu, L. Xiong and S. Zhang, *Smallest 2-edge-connected graphs without a spanning trail*, Util. Math. **88** (2012) 381–397.
- [10] Z. Ryjáček, *On a closure concept in claw-free graphs*, J. Combin. Theory Ser. B **70** (1997) 217–224.
doi:10.1006/jctb.1996.1732
- [11] S. Wang and L. Xiong, *Forbidden pairs for traceability of 2-connected graphs*, submitted.
- [12] J. Xu, P. Li, Z. Miao, K. Wang and H. Lai, *Supereulerian graphs with small matching number and 2-connected Hamiltonian claw-free graphs*, Int. J. Comput. Math. **91** (2014) 1662–1672.
doi:10.1080/00207160.2013.858808

Received 13 September 2016

Revised 12 December 2017

Accepted 30 December 2017