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ON THE INDEPENDENCE NUMBER OF TRACEABLE 2-CONNECTED CLAW-FREE GRAPHS

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Abstract

A well-known theorem by Chvátal-Erdős [*A note on Hamilton circuits*, Discrete Math. 2 (1972) 111–135] states that if the independence number of a graph G is at most its connectivity plus one, then G is traceable. In this article, we show that every 2-connected claw-free graph with independence number $\alpha(G) \leq 6$ is traceable or belongs to two exceptional families of well-defined graphs. As a corollary, we also show that every 2-connected claw-free graph with independence number $\alpha(G) \leq 5$ is traceable.

Keywords: traceability, independence number, matching number, trail, closure.

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