

BOUNDS ON THE LOCATING-TOTAL DOMINATION NUMBER IN TREES

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Abstract

Given a graph $G = (V, E)$ with no isolated vertex, a subset S of V is called a total dominating set of G if every vertex in V has a neighbor in S . A total dominating set S is called a locating-total dominating set if for each pair of distinct vertices u and v in $V \setminus S$, $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of a locating-total dominating set of G is the locating-total domination number, denoted by $\gamma_t^L(G)$. We show that, for a tree T of order $n \geq 3$ and diameter d , $\frac{d+1}{2} \leq \gamma_t^L(T) \leq n - \frac{d-1}{2}$, and if T has l leaves, s support vertices and s_1 strong support vertices, then $\gamma_t^L(T) \geq \max \left\{ \frac{n+l-s+1}{2} - \frac{s+s_1}{4}, \frac{2(n+1)+3(l-s)-s_1}{5} \right\}$. We also characterize the extremal trees achieving these bounds.

Keywords: tree, total dominating set, locating-total dominating set, locating-total domination number.

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