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THE TURÁN NUMBER OF $2P_7$

Yongxin Lan[1], Zhongmei Qin[2*]

and

Yongtang Shi[1]

[1] *Center for Combinatorics and LPMC*
Nankai University
Tianjin 300071, China

[2] *College of Science, Chang'an University*
Xi'an, Shaanxi 710064, China

e-mail: lan@mail.nankai.edu.cn
qinzhongmei90@163.com
shi@nankai.edu.cn

Abstract

The Turán number of a graph H, denoted by $ex(n, H)$, is the maximum number of edges in any graph on n vertices which does not contain H as a subgraph. Let P_k denote the path on k vertices and let mP_k denote m disjoint copies of P_k. Bushaw and Kettle [*Turán numbers of multiple paths and equibipartite forests*, Combin. Probab. Comput. 20 (2011) 837–853] determined the exact value of $ex(n, kP_\ell)$ for large values of n. Yuan and Zhang [*The Turán number of disjoint copies of paths*, Discrete Math. 340 (2017) 132–139] completely determined the value of $ex(n, kP_3)$ for all n, and also determined $ex(n, F_m)$, where F_m is the disjoint union of m paths containing at most one odd path. They also determined the exact value of $ex(n, P_3 \cup P_{2\ell+1})$ for $n \geq 2\ell + 4$. Recently, Bielak and Kieliszek [*The Turán number of the graph* $2P_5$, Discuss. Math. Graph Theory 36 (2016) 683–694], Yuan and Zhang [*Turán numbers for disjoint paths*, arXiv:1611.00981v1] independently determined the exact value of $ex(n, 2P_5)$. In this paper, we show that $ex(n, 2P_7) = \max\{[n, 14, 7], 5n - 14\}$ for all $n \geq 14$, where $[n, 14, 7] = (5n + 91 + r(r - 6))/2$, $n - 13 \equiv r \pmod 6$ and $0 \leq r < 6$.

Keywords: Turán number, extremal graphs, $2P_7$.

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*Corresponding author.

References

[1] P.N. Balister, E. Győri, J. Lehel and R.H. Schelp, *Connected graphs without long paths*, Discrete Math. **308** (2008) 4487–4494.
doi:10.1016/j.disc.2007.08.047

[2] H. Bielak and S. Kieliszek, *The Turán number of the graph* $2P_5$, Discuss. Math. Graph Theory **36** (2016) 683–694.
doi:10.7151/dmgt.1883

[3] B. Bollobás, Modern Graph Theory (Springer, 2013).

[4] N. Bushaw and N. Kettle, *Turán numbers of multiple paths and equibipartite forests*, Combin. Probab. Comput. **20** (2011) 837–853.
doi:10.1017/S0963548311000460

[5] P. Erdős and T. Gallai, *On maximal paths and circuits of graphs*, Acta Math. Acad. Sci. Hungar **10** (1959) 337–356.
doi:10.1007/BF02024498

[6] P. Erdős, *Über ein Extremalproblem in der Graphentheorie*, Arch. Math. (Basel) **13** (1962) 222–227, in German.
doi:10.1007/BF01650069

[7] R.J. Faudree and R.H. Schelp, *Path Ramsey numbers in multicolourings*, J. Combin. Theory Ser. B **19** (1975) 150–160.
doi:10.1016/0095-8956(75)90080-5

[8] G.N. Kopylov, *Maximal paths and cycles in a graph*, Dokl. Akad. Nauk SSSR **234** (1977) 19–21. (English translation in Soviet Math. Dokl. **18** (1977) 593–596.)

[9] Y. Lan, Y. Shi and Z.-X. Song, *Planar Turán numbers for Theta graphs and paths of small order*, arXiv:1711.01614v1.

[10] B. Lidický, H. Liu and C. Palmer, *On the Turán number of forests*, Electron. J. Combin. **20** (2013) #P62.

[11] J.W. Moon, *On independent complete subgraphs in a graph*, Canad. J. Math. **20** (1968) 95–102.
doi:10.4153/CJM-1968-012-x

[12] M. Simonovits, *A method for solving extremal problems in graph theory, stability problems*, in: Theory of Graphs (Academic Press, 1968) 279–319.

[13] P. Turán, *Eine Extremalaufgabe aus der Graphentheorie*, Mat. Fiz. Lapok **48** (1941) 436–452, in Hungarian.

[14] P. Turán, *On the theory of graphs*, Colloq. Math. **3** (1954) 19–30.
doi:10.4064/cm-3-1-19-30

[15] L. Yuan and X. Zhang, *Turán numbers for disjoint paths*, arXiv: 1611.00981v1.

[16] L. Yuan and X. Zhang, *The Turán number of disjoint copies of paths*, Discrete Math. **340** (2017) 132–139.
doi:10.1016/j.disc.2016.08.004

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