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TOTAL ROMAN REINFORCEMENT IN GRAPHS

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Abstract

A total Roman dominating function on a graph G is a labeling $f : V(G) \to \{0, 1, 2\}$ such that every vertex with label 0 has a neighbor with label 2 and the subgraph of G induced by the set of all vertices of positive weight has no isolated vertex. The minimum weight of a total Roman dominating function on a graph G is called the total Roman domination number of G. The total Roman reinforcement number $r_{tR}(G)$ of a graph G is the minimum number of edges that must be added to G in order to decrease the total Roman domination number. In this paper, we investigate the properties of total Roman reinforcement number in graphs, and we present some sharp bounds for $r_{tR}(G)$. Moreover, we show that the decision problem for total Roman reinforcement is NP-hard for bipartite graphs.

Keywords: total Roman domination number, total Roman reinforcement number.

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