

KALEIDOSCOPIC EDGE-COLORING OF COMPLETE GRAPHS AND r -REGULAR GRAPHS¹

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Abstract

For an r -regular graph G , we define an edge-coloring c with colors from $\{1, 2, \dots, k\}$, in such a way that any vertex of G is incident with at least one edge of each color. The multiset-color $c_m(v)$ of a vertex v is defined as the ordered tuple $(a_1, a_2, \dots, a_k)$, where a_i ($1 \leq i \leq k$) denotes the number of edges of color i which are incident with v in G . Then this edge-coloring c is called a k -kaleidoscopic coloring of G if every two distinct vertices in G have different multiset-colors and in this way the graph G is defined as a k -kaleidoscope. In this paper, we determine the integer k for a complete graph K_n to be a k -kaleidoscope, and hence solve a conjecture in [P. Zhang, A Kaleidoscopic View of Graph Colorings, (Springer Briefs in Math., New York, 2016)] that for any integers n and k with $n \geq k + 3 \geq 6$, the complete graph K_n is a k -kaleidoscope. Then, we construct an r -regular 3-kaleidoscope of order $\binom{r-1}{2} - 1$ for each integer $r \geq 7$, where $r \equiv 3 \pmod{4}$, which solves another conjecture in [P. Zhang, A Kaleidoscopic View of Graph Colorings, (Springer Briefs in Math., New York, 2016)] on the maximum order of r -regular 3-kaleidoscopes.

Keywords: k -kaleidoscope, regular graph, edge-coloring.

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