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KALEIDOSCOPIC EDGE-COLORING OF COMPLETE GRAPHS AND r-REGULAR GRAPHS[1]

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Abstract

For an r-regular graph G, we define an edge-coloring c with colors from $\{1, 2, \ldots, k\}$, in such a way that any vertex of G is incident with at least one edge of each color. The multiset-color $c_m(v)$ of a vertex v is defined as the ordered tuple $(a_1, a_2, \ldots, a_k)$, where a_i $(1 \leq i \leq k)$ denotes the number of edges of color i which are incident with v in G. Then this edge-coloring c is called a *k-kaleidoscopic coloring* of G if every two distinct vertices in G have different multiset-colors and in this way the graph G is defined as a *k-kaleidoscope*. In this paper, we determine the integer k for a complete graph K_n to be a k-kaleidoscope, and hence solve a conjecture in [P. Zhang, A Kaleidoscopic View of Graph Colorings, (Springer Briefs in Math., New York, 2016)] that for any integers n and k with $n \geq k + 3 \geq 6$, the complete graph K_n is a k-kaleidoscope. Then, we construct an r-regular 3-kaleidoscope of order $\binom{r-1}{2} - 1$ for each integer $r \geq 7$, where $r \equiv 3 \pmod 4$, which solves another conjecture in [P. Zhang, A Kaleidoscopic View of Graph Colorings, (Springer Briefs in Math., New York, 2016)] on the maximum order of r-regular 3-kaleidoscopes.

Keywords: k-kaleidoscope, regular graph, edge-coloring.

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