Discussiones Mathematicae
Graph Theory 39 (2019) 867–879
doi:10.7151/dmgt.2106

PANCYCLICITY WHEN EACH CYCLE CONTAINS k CHORDS

Vladislav Taranchuk

Department of Mathematics and Statistics
California State University, Sacramento
Sacramento, CA

e-mail: Vladislavt3@outlook.com

Abstract

For integers $n \geq k \geq 2$, let $c(n,k)$ be the minimum number of chords that must be added to a cycle of length n so that the resulting graph has the property that for every $l \in \{k, k+1, \ldots, n\}$, there is a cycle of length l that contains exactly k of the added chords. Affif Chaouche, Rutherford, and Whitty introduced the function $c(n,k)$. They showed that for every integer $k \geq 2$, $c(n,k) \geq \Omega_k(n^{1/k})$ and they asked if $n^{1/k}$ gives the correct order of magnitude of $c(n,k)$ for $k \geq 2$. Our main theorem answers this question as we prove that for every integer $k \geq 2$, and for sufficiently large n, $c(n,k) \leq k\lceil n^{1/k} \rceil + k^2$. This upper bound, together with the lower bound of Affif Chaouche *et al.*, shows that the order of magnitude of $c(n,k)$ is $n^{1/k}$.

Keywords: pancyclicity, chords.

2010 Mathematics Subject Classification: 05D99, 11B75.

References

[1] F. Affif Chaouche, C. Rutherford and R. Whitty, *Pancyclicity when each cycle must pass exactly k Hamilton cycle chords*, Discuss. Math. Graph Theory **35** (2015) 533–539.
doi:10.7151/dmgt.1818

[2] J.A. Bondy, *Pancyclic graphs* I, J. Combin. Theory Ser. B **11** (1971) 80–84.
doi:10.1016/0095-8956(71)90016-5

[3] H.J. Broersma, *A note on the minimum size of a vertex pancyclic graph*, Discrete Math. **164** (1997) 29–32.
doi:10.1016/S0012-365X(96)00040-4

[4] J.C. George, A. Khodkar and W.D. Wallis, Pancyclic and Bipancyclic Graphs, 1st Ed. (Springer Briefs in Mathematics, Springer, 2016).

[5] S. Griffin, *Minimal pancyclicity.*
arXiv: 1312.0274v1 1 Dec 2013.

[6] D.B. West, Introduction to Graph Theory, 2nd Edition (Pearson Education, Inc., 2001).

Received 26 September 2016
Revised 11 December 2017
Accepted 11 December 2017