

ON INDEPENDENT DOMINATION IN PLANAR CUBIC GRAPHS

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Abstract

A set S of vertices in a graph G is an independent dominating set of G if S is an independent set and every vertex not in S is adjacent to a vertex in S . The independent domination number, $i(G)$, of G is the minimum cardinality of an independent dominating set. Goddard and Henning [Discrete Math. 313 (2013) 839–854] posed the conjecture that if $G \notin \{K_{3,3}, C_5 \square K_2\}$ is a connected, cubic graph on n vertices, then $i(G) \leq \frac{3}{8}n$, where $C_5 \square K_2$ is the 5-prism. As an application of known result, we observe that this conjecture is true when G is 2-connected and planar, and we provide an infinite family of such graphs that achieve the bound. We conjecture that if G is a bipartite, planar, cubic graph of order n , then $i(G) \leq \frac{1}{3}n$, and we provide an infinite family of such graphs that achieve this bound.

Keywords: independent domination number, domination number, cubic graphs.

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