

ABOUT (k, l) -KERNELS, SEMIKERNELS AND GRUNDY FUNCTIONS IN PARTIAL LINE DIGRAPHS¹

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Abstract

Let D be a digraph of minimum in-degree at least 1. We prove that for any two natural numbers k, l such that $1 \leq l \leq k$, the number of (k, l) -kernels of D is less than or equal to the number of (k, l) -kernels of any partial line digraph $\mathcal{L}D$. Moreover, if $l < k$ and the girth of D is at least $l + 1$, then these two numbers are equal. We also prove that the number of semikernels of D is equal to the number of semikernels of $\mathcal{L}D$. Furthermore, we introduce the concept of (k, l) -Grundy function as a generalization of the concept of

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Grundy function and we prove that the number of (k, l) -Grundy functions of D is equal to the number of (k, l) -Grundy functions of any partial line digraph $\mathcal{LD}$.

Keywords: digraphs, in-domination, kernel, Grundy function.

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