Discussiones Mathematicae
Graph Theory 39 (2019) 855–866
doi:10.7151/dmgt.2104

ABOUT (k, l)-KERNELS, SEMIKERNELS AND GRUNDY FUNCTIONS IN PARTIAL LINE DIGRAPHS[1]

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Abstract

Let D be a digraph of minimum in-degree at least 1. We prove that for any two natural numbers k, l such that $1 \leq l \leq k$, the number of (k, l)-kernels of D is less than or equal to the number of (k, l)-kernels of any partial line digraph $\mathcal{L}D$. Moreover, if $l < k$ and the girth of D is at least $l+1$, then these two numbers are equal. We also prove that the number of semikernels of D is equal to the number of semikernels of $\mathcal{L}D$. Furthermore, we introduce the concept of (k, l)-Grundy function as a generalization of the concept of

[1]This research was supported by the Ministerio de Educación y Ciencia, Spain, the European Regional Development Fund (ERDF) under project MTM2014-60127-P, by CONACyT-México under project 219840 and by UNAM-DGAPA-PAPIIT IN104717 and IN115816.

Grundy function and we prove that the number of (k,l)-Grundy functions of D is equal to the number of (k,l)-Grundy functions of any partial line digraph $\mathcal{L}D$.

Keywords: digraphs, in-domination, kernel, Grundy function.

2010 Mathematics Subject Classification: 05C20, 05C69, 05C76.

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Received 28 June 2016
Revised 9 December 2017
Accepted 11 December 2017