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DOMINATION SUBDIVISION AND DOMINATION MULTISUBDIVISION NUMBERS OF GRAPHS

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Abstract

The *domination subdivision number* $\mathrm{sd}(G)$ of a graph G is the minimum number of edges that must be subdivided (where an edge can be subdivided at most once) in order to increase the domination number of G. It has been shown [10] that $\mathrm{sd}(T) \leq 3$ for any tree T. We prove that the decision problem of the domination subdivision number is NP-complete even for bipartite graphs. For this reason we define the *domination multisubdivision number* of a nonempty graph G as a minimum positive integer k such that there exists an edge which must be subdivided k times to increase the domination number of G. We show that $\mathrm{msd}(G) \leq 3$ for any graph G. The domination subdivision number and the domination multisubdivision number of a graph are incomparable in general, but we show that for trees these two parameters are equal. We also determine the domination multisubdivision number for some classes of graphs.

Keywords: domination, domination subdivision number, domination multisubdivision number, trees, computational complexity.

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