Discussiones Mathematicae
Graph Theory 39 (2019) 575–588
doi:10.7151/dmgt.2102

1-RESTRICTED OPTIMAL RUBBLING ON GRAPHS

Robert A. Beeler

Department of Mathematics and Statistics
East Tennessee State University
Johnson City, TN 37614-1700 USA

e-mail: beelerr@etsu.edu

Teresa W. Haynes

Department of Mathematics and Statistics
East Tennessee State University
Johnson City, TN 37614-1700 USA
Department of Mathematics
University of Johannesburg
Auckland Park, South Africa

e-mail: haynes@etsu.edu

AND

Kyle Murphy

Department of Mathematics
Iowa State University
Ames, IA 50011 USA

e-mail: kylem2@iastate.edu

Abstract

Let G be a graph with vertex set V and a distribution of pebbles on the vertices of V. A pebbling move consists of removing two pebbles from a vertex and placing one pebble on a neighboring vertex, and a rubbling move consists of removing a pebble from each of two neighbors of a vertex v and placing a pebble on v. We seek an initial placement of a minimum total number of pebbles on the vertices in V, so that no vertex receives more than one pebble and for any given vertex $v \in V$, it is possible, by a sequence of pebbling and rubbling moves, to move at least one pebble to v. This minimum number of pebbles is the 1-restricted optimal rubbling number. We determine the 1-restricted optimal rubbling numbers for Cartesian products. We also present bounds on the 1-restricted optimal rubbling number.

Keywords: graph pebbling, graph rubbling, optimal rubbling, t-restricted optimal pebbling.

2010 Mathematics Subject Classification: 05C78.

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Received 1 May 2017
Revised 23 October 2017
Accepted 24 October 2017