

1-RESTRICTED OPTIMAL RUBBLING ON GRAPHS

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Abstract

Let G be a graph with vertex set V and a distribution of pebbles on the vertices of V . A pebbling move consists of removing two pebbles from a vertex and placing one pebble on a neighboring vertex, and a rubbling move consists of removing a pebble from each of two neighbors of a vertex v and placing a pebble on v . We seek an initial placement of a minimum total number of pebbles on the vertices in V , so that no vertex receives more than one pebble and for any given vertex $v \in V$, it is possible, by a sequence of pebbling and rubbling moves, to move at least one pebble to v . This minimum number of pebbles is the 1-restricted optimal rubbling number. We determine the 1-restricted optimal rubbling numbers for Cartesian products. We also present bounds on the 1-restricted optimal rubbling number.

Keywords: graph pebbling, graph rubbing, optimal rubbing, t -restricted optimal pebbling.

2010 Mathematics Subject Classification: 05C78.

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Received 1 May 2017

Revised 23 October 2017

Accepted 24 October 2017