

## ON DECOMPOSING THE COMPLETE SYMMETRIC DIGRAPH INTO ORIENTATIONS OF $K_4 - e$

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### Abstract

Let  $D$  be any of the 10 digraphs obtained by orienting the edges of  $K_4 - e$ . We establish necessary and sufficient conditions for the existence of a  $(K_n^*, D)$ -design for 8 of these digraphs. Partial results as well as some nonexistence results are established for the remaining 2 digraphs.

**Keywords:** digraph decompositions, orientations of  $K_4 - e$ .

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