

## ON THE TOTAL ROMAN DOMINATION IN TREES

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### Abstract

A *total Roman dominating function* on a graph  $G$  is a function  $f : V(G) \rightarrow \{0, 1, 2\}$  satisfying the following conditions: (i) every vertex  $u$  for which  $f(u) = 0$  is adjacent to at least one vertex  $v$  for which  $f(v) = 2$  and (ii) the subgraph of  $G$  induced by the set of all vertices of positive weight has no isolated vertex. The weight of a total Roman dominating function  $f$  is the value  $f(V(G)) = \sum_{u \in V(G)} f(u)$ . The *total Roman domination number*  $\gamma_{tR}(G)$  is the minimum weight of a total Roman dominating function of  $G$ . Ahangar *et al.* in [H.A. Ahangar, M.A. Henning, V. Samodivkin and I.G. Yero, *Total Roman domination in graphs*, Appl. Anal. Discrete Math. 10 (2016) 501–517] recently showed that for any graph  $G$  without isolated vertices,  $2\gamma(G) \leq \gamma_{tR}(G) \leq 3\gamma(G)$ , where  $\gamma(G)$  is the domination number of  $G$ , and they raised the problem of characterizing the graphs  $G$  achieving these upper and lower bounds. In this paper, we provide a constructive characterization of these trees.

**Keywords:** total Roman dominating function, total Roman domination number, trees.

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