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SUFFICIENT CONDITIONS FOR MAXIMALLY EDGE-CONNECTED AND SUPER-EDGE-CONNECTED GRAPHS DEPENDING ON THE CLIQUE NUMBER

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Abstract

Let G be a connected graph with minimum degree δ and edge-connectivity λ. A graph is maximally edge-connected if $\lambda = \delta$, and it is super-edge-connected if every minimum edge-cut is trivial; that is, if every minimum edge-cut consists of edges incident with a vertex of minimum degree. The clique number $\omega(G)$ of a graph G is the maximum cardinality of a complete subgraph of G. In this paper, we show that a connected graph G with clique number $\omega(G) \leq r$ is maximally edge-connected or super-edge-connected if the number of edges is large enough. These are generalizations of corresponding results for triangle-free graphs by Volkmann and Hong in 2017.

Keywords: edge-connectivity, clique number, maximally edge-connected graphs, super-edge-connected graphs.

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