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ERDŐS-GALLAI-TYPE RESULTS FOR TOTAL MONOCHROMATIC CONNECTION OF GRAPHS

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Abstract

A graph is said to be *total-colored* if all the edges and the vertices of the graph are colored. A total-coloring of a graph is a *total monochromatically-connecting coloring* (TMC-coloring, for short) if any two vertices of the graph are connected by a path whose edges and internal vertices have the same color. For a connected graph G, the *total monochromatic connection number*, denoted by $tmc(G)$, is defined as the maximum number of colors used in a TMC-coloring of G. In this paper, we study two kinds of Erdős-Gallai-type problems for $tmc(G)$ and completely solve them.

Keywords: total-colored graph, total monochromatic connection, Erdős-Gallai-type problem.

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