Discussiones Mathematicae
Graph Theory 39 (2019) 533–546
doi:10.7151/dmgt.2094

ORIENTABLE $\mathbb{Z}_N$-DISTANCE MAGIC GRAPHS

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Abstract

Let $G = (V, E)$ be a graph of order n. A distance magic labeling of G is a bijection $\ell\colon V \to \{1, 2, \ldots, n\}$ for which there exists a positive integer k such that $\sum_{x \in N(v)} \ell(x) = k$ for all $v \in V$, where $N(v)$ is the open neighborhood of v.

Tuttes flow conjectures are a major source of inspiration in graph theory. In this paper we ask when we can assign n distinct labels from the set $\{1, 2, \ldots, n\}$ to the vertices of a graph G of order n such that the sum of the labels on heads minus the sum of the labels on tails is constant modulo n for each vertex of G. Therefore we generalize the notion of distance magic labeling for oriented graphs.

Keywords: distance magic graph, digraph, flow graph.

2010 Mathematics Subject Classification: 5C15, 05C22, 05C25, 05C76, 05C78.

[1]This work was partially supported by the Faculty of Applied Mathematics AGH UST statutory tasks within subsidy of Ministry of Science and Higher Education.

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Received 14 March 2017
Revised 28 September 2017
Accepted 28 September 2017