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ON THE CO-ROMAN DOMINATION IN GRAPHS

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Abstract

Let $G = (V, E)$ be a graph and let $f : V(G) \to \{0, 1, 2\}$ be a function. A vertex v is said to be protected with respect to f, if $f(v) > 0$ or $f(v) = 0$ and v is adjacent to a vertex of positive weight. The function f is a *co-Roman dominating function* if (i) every vertex in V is protected, and (ii) each $v \in V$ with positive weight has a neighbor $u \in V$ with $f(u) = 0$ such that the function $f_{uv} : V \to \{0, 1, 2\}$, defined by $f_{uv}(u) = 1$, $f_{uv}(v) = f(v) - 1$ and $f_{uv}(x) = f(x)$ for $x \in V \setminus \{v, u\}$, has no unprotected vertex. The *weight* of f is $\omega(f) = \sum_{v \in V} f(v)$. The *co-Roman domination number* of a graph G, denoted by $\gamma_{cr}(G)$, is the minimum weight of a co-Roman dominating function on G. In this paper, we give a characterization of graphs of order n for which co-Roman domination number is $\frac{2n}{3}$ or $n - 2$, which settles

two open problem in [S. Arumugam, K. Ebadi and M. Manrique, *Co-Roman domination in graphs*, Proc. Indian Acad. Sci. Math. Sci. 125 (2015) 1–10]. Furthermore, we present some sharp bounds on the co-Roman domination number.

Keywords: co-Roman dominating function, co-Roman domination number, Roman domination.

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