

## ARANKINGS OF TREES

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### Abstract

For a graph  $G = (V, E)$ , a function  $f : V(G) \rightarrow \{1, 2, \dots, k\}$  is a  $k$ -ranking for  $G$  if  $f(u) = f(v)$  implies that every  $u - v$  path contains a vertex  $w$  such that  $f(w) > f(u)$ . A minimal  $k$ -ranking,  $f$ , of a graph,  $G$ , is a  $k$ -ranking with the property that decreasing the label of any vertex results in the ranking property being violated. The rank number  $\chi_r(G)$  and the arank number  $\psi_r(G)$  are, respectively, the minimum and maximum value of  $k$  such that  $G$  has a minimal  $k$ -ranking. This paper establishes an upper bound for  $\psi_r$  of a tree and shows the bound is sharp for perfect  $k$ -ary trees.

**Keywords:** minimal ranking, coloring, tree.

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### REFERENCES

- [1] H. Alpert, *Rank numbers of grid graphs*, Discrete Math. **310** (2010) 3324–3333.  
doi:10.1016/j.disc.2010.07.022
- [2] H.L. Bodlaender, J.R. Gilbert, H. Hafsteinsson and T. Kloks, *Approximating treewidth, pathwidth, frontsize, and shortest elimination tree*, J. Algorithms **18** (1995) 238–255.  
doi:10.1006/jagm.1995.1009
- [3] H.L. Bodlaender, J.S. Deogun, K. Jansen, T. Kloks, D. Kratsch, H. Muller and Zs. Tuza, *Rankings of graphs*, SIAM J. Discrete Math. **11** (1998) 168–181.  
doi:10.1137/S0895480195282550
- [4] G. Chartrand and P. Zhang, *A First Course in Graph Theory* (Dover Books, New York, 2012).
- [5] P. de la Torre, R. Greenlaw and A.A. Schaeffer, *Optimal ranking of trees in polynomial time*, in: Proc. 4th ACM Symp. on Discrete Algorithms, V. Ramachandran (Ed(s)), (Austin, Texas, Society for Industrial and Applied Mathematics, 1993) 138–144.

- [6] J.S. Deogun, T. Kloks, D. Kratsch and H. Muller, *On vertex ranking for permutation and other graphs*, in: Lecture Notes in Comput. Sci. **775**, P. Enjalbert, E.W. Mayr and K.W. Wagner (Ed(s)), (Berlin, Springer-Verlag, 1994) 747–758.  
doi:10.1007/3-540-57785-8\_187
- [7] I.S. Duff and J.K. Reid, *The multifrontal solution of indefinite sparse symmetric linear equations*, ACM Trans. Math. Software **9** (1983) 302–325.  
doi:10.1145/356044.356047
- [8] D.R. Fulkerson and O.A. Gross, *Incidence matrices and interval graphs*, Pacific J. Math. **15** (1965) 835–855.  
doi:10.2140/pjm.1965.15.835
- [9] J. Ghoshal, R. Laskar and D. Pillone, *Minimal rankings*, Networks **28** (1996) 45–53.  
doi:10.1002/(SICI)1097-0037(199608)28:1<45::AID-NET6>3.0.CO;2-D
- [10] J. Ghoshal, R. Laskar and D. Pillone, *Further results on minimal rankings*, Ars Combin. **52** (1999) 191–198.
- [11] M. Katchalski, W. McCuaig and S. Seager, *Ordered colourings*, Discrete Math. **142** (1995) 141–154.  
doi:10.1016/0012-365X(93)E0216-Q
- [12] R. Laskar, and D. Pillone, *Extremal results in rankings*, Congr. Numer. **149** (2001) 33–54.
- [13] R. Laskar, D. Pillone, J. Jacob, G. Eyabi and D. Narayan, *Minimal rankings of the Cartesian product  $K_m \square K_n$* , Discuss. Math. Graph Theory **32** (2012) 725–735.  
doi:10.7151/dmgt.1634
- [14] R. Laskar and D. Pillone, *Theoretical and complexity results for minimal rankings*, J. Comb. Inf. Syst. Sci. **25** (2000) 17–33.
- [15] J.W.H. Liu, *The role of elimination trees in sparse factorization*, SIAM J. Matrix Anal. Appl. **11** (1990) 134–172.  
doi:10.1137/0611010
- [16] V. Kostyuk, D.A. Narayan and V.A. Williams, *Minimal rankings and the arank number of a path*, Discrete Math. **306** (2006) 1991–1996.  
doi:10.1016/j.disc.2006.01.027

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