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FAIR DOMINATION NUMBER IN CACTUS GRAPHS

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Abstract

For $k \geq 1$, a k-fair dominating set (or just kFD-set) in a graph G is a dominating set S such that $\left|N(v) \cap S\right| = k$ for every vertex $v \in V \setminus S$. The k-fair domination number of G, denoted by $fd_k(G)$, is the minimum cardinality of a kFD-set. A fair dominating set, abbreviated FD-set, is a kFD-set for some integer $k \geq 1$. The fair domination number, denoted by $fd(G)$, of G that is not the empty graph, is the minimum cardinality of an FD-set in G. In this paper, aiming to provide a particular answer to a problem posed in [Y. Caro, A. Hansberg and M.A. Henning, *Fair domination in graphs*, Discrete Math. 312 (2012) 2905–2914], we present a new upper bound for the fair domination number of a cactus graph, and characterize all cactus graphs G achieving equality in the upper bound of $fd_1(G)$.

Keywords: fair domination, cactus graph, unicyclic graph.

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