

## ERROR-CORRECTING CODES FROM $k$ -RESOLVING SETS

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### Abstract

We demonstrate a construction of error-correcting codes from graphs by means of  $k$ -resolving sets, and present a decoding algorithm which makes use of covering designs. Along the way, we determine the  $k$ -metric dimension of grid graphs (i.e., Cartesian products of paths).

**Keywords:** error-correcting code,  $k$ -resolving set,  $k$ -metric dimension, covering design, uncovering, grid graph.

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### REFERENCES

- [1] R.F. Bailey, *Uncoverings-by-bases for base-transitive permutation groups*, Des. Codes Cryptogr. **41** (2006) 153–176.  
doi:10.1007/s10623-006-9005-x
- [2] R.F. Bailey, *Error-correcting codes from permutation groups*, Discrete Math. **309** (2009) 4253–4265.  
doi:10.1016/j.disc.2008.12.027

- [3] R.F. Bailey and P.J. Cameron, *Base size, metric dimension and other invariants of groups and graphs*, Bull. Lond. Math. Soc. **43** (2011) 209–242.  
doi:10.1112/blms/bdq096
- [4] R.F. Bailey and B. Stevens, *Uncoverings on graphs and network reliability*, Australas. J. Combin. **50** (2011) 219–231.
- [5] A.F. Beardon and J.A. Rodríguez-Velázquez, *On the  $k$ -metric dimension of metric spaces*, preprint.  
arXiv:1603.04049.
- [6] L.M. Blumenthal, *Theory and Applications of Distance Geometry* (Clarendon Press, Oxford, 1953).
- [7] P.J. Cameron, *Permutation codes*, European J. Combin. **31** (2010) 482–490.  
doi:10.1016/j.ejc.2009.03.044
- [8] G. Chartrand, L. Eroh, M.A. Johnson and O.R. Oellermann, *Resolvability in graphs and the metric dimension of a graph*, Discrete Appl. Math. **105** (2000) 99–113.  
doi:10.1016/S0166-218X(00)00198-0
- [9] W. Chu, C.J. Colbourn and P. Dukes, *Constructions for permutation codes in powerline communications*, Des. Codes Cryptogr. **32** (2004) 51–64.  
doi:10.1023/B:DESI.0000029212.52214.71
- [10] C.J. Colbourn and J.H. Dinitz (Editors), *Handbook of Combinatorial Designs* (Second Edition) (CRC Press, Boca Raton, 2007).
- [11] T.H. Cormen, C.E. Leiserson, R.L. Rivest and C. Stein, *Introduction to Algorithms* (Second Edition) (MIT Press, Cambridge/McGraw-Hill, Boston, 2001).
- [12] A. Estrada-Moreno, J.A. Rodríguez-Velázquez and I.G. Yero, *The  $k$ -metric dimension of a graph*, Appl. Math. Inf. Sci. **9** (2015) 2829–2840.
- [13] A. Estrada-Moreno, I.G. Yero and J.A. Rodríguez-Velázquez, *The  $k$ -metric dimension of corona product graphs*, Bull. Malays. Math. Sci. Soc. **39** (2016) 135–156.  
doi:10.1007/s40840-015-0282-2
- [14] A. Estrada-Moreno, I.G. Yero and J.A. Rodríguez-Velázquez, *The  $k$ -metric dimension of the lexicographic product of graphs*, Discrete Math. **339** (2016) 1924–1934.  
doi:10.1016/j.disc.2015.12.024
- [15] D.M. Gordon, La Jolla Covering Repository.  
www.ccrwest.org/cover.html
- [16] F. Harary and R.A. Melter, *On the metric dimension of a graph*, Ars Combin. **2** (1976) 191–195.
- [17] S. Khuller, B. Raghavachari and A. Rosenfeld, *Landmarks in graphs*, Discrete Appl. Math. **70** (1996) 217–229.  
doi:10.1016/0166-218X(95)00106-2
- [18] H.-J. Kroll and R. Vincenti, *Antiblocking systems and PD-sets*, Discrete Math. **308** (2008) 401–407.  
doi:10.1016/j.disc.2006.11.056

- [19] W. H. Mills and R.C. Mullin, *Coverings and packings*, in: Contemporary Design Theory: A collection of surveys, J.H. Dinitz and D.R. Stinson (Ed(s)), (John Wiley & Sons, New York, 1992) 371–399.
- [20] V. Pless, Introduction to the Theory of Error-Correcting Codes (Third Edition) (John Wiley & Sons, New York, 1998).  
doi:10.1002/9781118032749
- [21] J. Schönheim, *On coverings*, Pacific J. Math. **14** (1964) 1405–1411.  
doi:10.2140/pjm.1964.14.1405
- [22] A. Sebő and E. Tannier, *On metric generators of graphs*, Math. Oper. Res. **29** (2004) 383–393.  
doi:10.1287/moor.1030.0070
- [23] P.J. Slater, *Leaves of trees*, Congr. Numer. **14** (1975) 549–559.
- [24] I. Tamo and M. Schwartz, *Correcting limited-magnitude errors in the rank-modulation scheme*, IEEE Trans. Inform. Theory **56** (2010) 2551–2560.  
doi:10.1109/TIT.2010.2046241
- [25] I.G. Yero, A. Estrada-Moreno and J.A. Rodríguez-Velázquez, *Computing the  $k$ -metric dimension of graphs*, Appl. Math. Comput. **300** (2017) 60–69.  
doi:10.1016/j.amc.2016.12.005

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