

## $L(2, 1)$ -LABELING OF CIRCULANT GRAPHS

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### Abstract

An  $L(2, 1)$ -labeling of a graph  $\Gamma$  is an assignment of non-negative integers to the vertices such that adjacent vertices receive labels that differ by at least 2, and those at a distance of two receive labels that differ by at least one. Let  $\lambda_2^1(\Gamma)$  denote the least  $\lambda$  such that  $\Gamma$  admits an  $L(2, 1)$ -labeling using labels from  $\{0, 1, \dots, \lambda\}$ . A Cayley graph of group  $G$  is called a circulant graph of order  $n$ , if  $G = \mathbb{Z}_n$ . In this paper initially we investigate the upper bound for the span of the  $L(2, 1)$ -labeling for Cayley graphs on cyclic groups with “large” connection sets. Then we extend our observation and find the span of  $L(2, 1)$ -labeling for any circulants of order  $n$ .

**Keywords:** graph coloring,  $L(2, 1)$ -labeling, circulants.

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