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$L(2,1)$-LABELING OF CIRCULANT GRAPHS

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Abstract

An $L(2,1)$-labeling of a graph Γ is an assignment of non-negative integers to the vertices such that adjacent vertices receive labels that differ by at least 2, and those at a distance of two receive labels that differ by at least one. Let $\lambda_2^1(\Gamma)$ denote the least λ such that Γ admits an $L(2,1)$-labeling using labels from $\{0,1,\ldots,\lambda\}$. A Cayley graph of group G is called a circulant graph of order n, if $G = \mathbb{Z}_n$. In this paper initially we investigate the upper bound for the span of the $L(2,1)$-labeling for Cayley graphs on cyclic groups with "large" connection sets. Then we extend our observation and find the span of $L(2,1)$-labeling for any circulants of order n.

Keywords: graph coloring, $L(2,1)$-labeling, circulants.

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