

ON 3-COLORINGS OF DIRECT PRODUCTS OF GRAPHS

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Abstract

The k -independence number of a graph G , denoted as $\alpha_k(G)$, is the order of a largest induced k -colorable subgraph of G . In [S. Špacapan, *The k -independence number of direct products of graphs*, European J. Combin. 32 (2011) 1377–1383] the author conjectured that the direct product $G \times H$ of graphs G and H obeys the following bound

$$\alpha_k(G \times H) \leq \alpha_k(G)|V(H)| + \alpha_k(H)|V(G)| - \alpha_k(G)\alpha_k(H),$$

and proved the conjecture for $k = 1$ and $k = 2$. If true for $k = 3$ the conjecture strenghtens the result of El-Zahar and Sauer who proved that any direct product of 4-chromatic graphs is 4-chromatic [M. El-Zahar and N. Sauer, *The chromatic number of the product of two 4-chromatic graphs is 4*, Combinatorica 5 (1985) 121–126]. In this paper we prove that the above bound is true for $k = 3$ provided that G and H are graphs that have complete tripartite subgraphs of orders $\alpha_3(G)$ and $\alpha_3(H)$, respectively.

Keywords: independence number, direct product, Hedetniemi's conjecture.

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REFERENCES

- [1] N. Alon, I. Dinur, E. Friedgut and B. Sudakov, *Graph products, Fourier analysis and spectral techniques*, Geom. Funct. Anal. **14** (2004) 913–940.
doi:10.1007/s00039-004-0478-3
- [2] M. El-Zahar and N. Sauer, *The chromatic number of the product of two 4-chromatic graphs is 4*, Combinatorica **5** (1985) 121–126.
doi:10.1007/BF02579374

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- [3] D. Greenwell and L. Lovász, *Applications of product colouring*, Acta Math. Acad. Sci. Hungar. **25** (1974) 335–340.
doi:10.1007/BF01886093
- [4] X.B. Geng, J. Wang and H. Zhang, *Structure of independent sets in direct products of some vertex-transitive graphs*, Acta Math. Sin. (Engl. Ser.) **28** (2012) 697–706.
doi:10.1007/s10114-011-0311-5
- [5] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs*, Second Edition (CRC Press, Boca Raton, FL, 2011).
- [6] S.T. Hedetniemi, *Homomorphisms and graph automata*, University of Michigan Technical Report 03105–44–T (1966).
- [7] P.K. Jha and S. Klavžar, *Independence in direct-product graphs*, Ars Combin. **50** (1998) 53–63.
- [8] B. Larose and C. Tardif, *Projectivity and independent sets in powers of graphs*, J. Graph Theory **40** (2002) 162–171.
doi:10.1002/jgt.10041
- [9] S. Špacapan, *The k -independence number of direct products of graphs and Hedetniemi’s conjecture*, European J. Combin **32** (2011) 1377–1383.
doi:10.1016/j.ejc.2011.07.002
- [10] C. Tardif, *Hedetniemi’s conjecture 40 years later*, Graph Theory Notes of New York LIV (2008) 46–57.
- [11] C. Tardif, *Hedetniemi’s conjecture and dense Boolean lattices*, Order **28** (2011) 181–191.
doi:10.1007/s11083-010-9162-4
- [12] Á. Tóth, *On the ultimate categorical independence ratio*, J. Combin. Theory Ser. B **108** (2014) 29–39.
doi:10.1016/j.jctb.2014.02.010
- [13] M. Valencia-Pabon and J. Vera, *Independence and coloring properties of direct products of some vertex-transitive graphs*, Discrete Math. **306** (2006) 2275–2281.
doi:10.1016/j.disc.2006.04.013
- [14] H. Zhang, *Primitivity and independent sets in direct products of vertex-transitive graphs*, J. Graph Theory **67** (2011) 218–225.
doi:10.1002/jgt.20526
- [15] H. Zhang, *Independent sets in direct products of vertex-transitive graphs*, J. Combin. Theory Ser. B **102** (2012) 832–838.
doi:10.1016/j.jctb.2011.12.005
- [16] X. Zhu, *A survey on Hedetniemi’s conjecture*, Taiwanese J. Math. **2** (1998) 1–24.
doi:10.11650/twjm/1500406890
- [17] X. Zhu, *The fractional version of Hedetniemi’s conjecture is true*, European J. Combin. **32** (2011) 1168–1175.
doi:10.1016/j.ejc.2011.03.004

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