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ON THE HAMILTONIAN NUMBER OF A PLANE GRAPH

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Abstract

The Hamiltonian number of a connected graph is the minimum of the lengths of the closed spanning walks in the graph. In 1968, Grinberg published a necessary condition for the existence of a Hamiltonian cycle in a plane graph, formulated in terms of the degrees of its faces. We show how Grinberg's theorem can be adapted to provide a lower bound on the Hamiltonian number of a plane graph.

Keywords: Hamiltonian cycle, Hamiltonian walk, Hamiltonian number, Hamiltonian spectrum, Grinberg's theorem, planar graph.

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