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ON SOME CHARACTERIZATIONS OF ANTIPODAL PARTIAL CUBES

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Abstract

We prove that any harmonic partial cube is antipodal, which was conjectured by Fukuda and K. Handa, *Antipodal graphs and oriented matroids*, Discrete Math. 111 (1993) 245–256. Then we prove that a partial cube G is antipodal if and only if the subgraphs induced by W_{ab} and W_{ba} are isomorphic for every edge ab of G. This gives a positive answer to a question of Klavžar and Kovše, *On even and harmonic-even partial cubes*, Ars Combin. 93 (2009) 77–86. Finally we prove that the distance-balanced partial cube that are antipodal are those whose pre-hull number is at most 1.

Keywords: diametrical graph, harmonic graph, antipodal graph, distance-balanced graph, partial cube, pre-hull number.

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