

ANTI-RAMSEY NUMBER OF HANOI GRAPHS

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Abstract

Let $\text{ar}(G, H)$ be the largest number of colors such that there exists an edge coloring of G with $\text{ar}(G, H)$ colors such that each subgraph isomorphic to H has at least two edges in the same color. We call $\text{ar}(G, H)$ the *anti-Ramsey number* for a pair of graphs (G, H) . This notion was introduced by Erdős, Simonovits and Sós in 1973 and studied in numerous papers.

Hanoi graphs were introduced by Scorer, Grundy and Smith in 1944 as the model of the well known Tower of Hanoi puzzle.

In the paper we study the anti-Ramsey number of Hanoi graphs and consider them both as the graph G and H . Among others we present the exact value of the anti-Ramsey number in case when both graphs are constructed for the same number of pegs.

Keywords: anti-Ramsey number, rainbow number, Hanoi graph.

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