

ORIENTED CHROMATIC NUMBER OF CARTESIAN PRODUCTS AND STRONG PRODUCTS OF PATHS

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Abstract

An oriented coloring of an oriented graph G is a homomorphism from G to H such that H is without selfloops and arcs in opposite directions. We shall say that H is a *coloring graph*. In this paper, we focus on oriented colorings of Cartesian products of two paths, called grids, and strong products of two paths, called strong-grids. We show that there exists a coloring graph with nine vertices that can be used to color every orientation of grids with five columns. We also show that there exists a strong-grid with two columns and its orientation which requires 11 colors for oriented coloring. Moreover, we show that every orientation of every strong-grid with three columns can be colored by 19 colors and that every orientation of every strong-grid with four columns can be colored by 43 colors. The above statements were proved with the help of computer programs.

Keywords: graph, oriented coloring, grid.

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REFERENCES

- [1] N.R. Aravind, N. Narayanan and C.R. Subramanian, *Oriented colouring of some graph products*, Discuss. Math. Graph Theory **31** (2011) 675–686.
doi:10.7151/dmgt.1572
- [2] O.V. Borodin, A.V. Kostochka, J. Nešetřil, A. Raspaud and É. Sopena, *On the maximum average degree and the oriented chromatic number of a graph*, Discrete Math. **206** (1999) 77–89.
doi:10.1016/S0012-365X(98)00393-8

- [3] J. Dybizbański and A. Nenca, *Oriented chromatic number of grids is greater than 7*, Inform. Process. Lett. **112** (2012) 113–117.
doi:10.1016/j.ipl.2011.10.019
- [4] J. Dybizbański and A. Szepietowski, *The oriented chromatic number of Halin graphs*, Inform. Process. Lett. **114** (2014) 45–49.
doi:10.1016/j.ipl.2013.09.011
- [5] G. Fertin, A. Raspaud and A. Roychowdhury, *On the oriented chromatic number of grids*, Inform. Process. Lett. **85** (2003) 261–266.
doi:10.1016/S0020-0190(02)00405-2
- [6] E. Fried, *On homogeneous tournaments*, Combin. Theory Appl. **2** (1970) 467–476.
- [7] A.V. Kostochka, É. Sopena and X. Zhu, *Acyclic and oriented chromatic numbers of graphs*, J. Graph Theory **24** (1997) 331–340.
doi:10.1002/(SICI)1097-0118(199704)24:4<331::AID-JGT5>3.0.CO;2-P
- [8] M. Hosseini Dolama and É. Sopena, *On the oriented chromatic number of graphs with given excess*, Discrete Math. **306** (2006) 1342–1350.
doi:10.1016/j.disc.2005.12.023
- [9] M. Hosseini Dolama and É. Sopena, *On the oriented chromatic number of Halin graphs*, Inform. Process. Lett. **98** (2006) 247–252.
doi:10.1016/j.ipl.2005.03.016
- [10] B.D. McKay and A. Piperno, *Practical graph isomorphism, II*, J. Symbolic Comput. **60** (2013) 94–112.
doi:10.1016/j.jsc.2013.09.003
- [11] A. Raspaud and É. Sopena, *Good and semi-strong colorings of oriented planar graphs*, Inform. Process. Lett. **51** (1994) 171–174.
doi:10.1016/0020-0190(94)00088-3
- [12] É. Sopena, *Homomorphisms and colourings of oriented graphs: An updated survey*, Discrete Math. **339** (2016) 1993–2005.
doi:10.1016/j.disc.2015.03.018
- [13] É. Sopena, *Oriented graph coloring*, Discrete Math. **229** (2001) 359–369.
doi:10.1016/S0012-365X(00)00216-8
- [14] É. Sopena, *The chromatic number of oriented graphs*, J. Graph Theory **25** (1997) 191–205.
doi:10.1002/(SICI)1097-0118(199707)25:3<191::AID-JGT3>3.0.CO;2-G
- [15] É. Sopena, *There exist oriented planar graphs with oriented chromatic number at least sixteen*, Inform. Process. Lett. **81** (2002) 309–312.
doi:10.1016/S0020-0190(01)00246-0
- [16] É. Sopena, *Upper oriented chromatic number of undirected graphs and oriented colorings of product graphs*, Discuss. Math. Graph Theory **32** (2012) 517–533.
doi:10.7151/dmgt.1624

- [17] É. Sopena and L. Vignal, *A note on the oriented chromatic number of graphs with maximum degree three*, Research Report (Bordeaux I University, 1996).
- [18] A. Szepietowski and M. Targan, *A note on the oriented chromatic number of grids*, Inform. Process. Lett. **92** (2004) 65–70.
doi:10.1016/j.ipl.2004.06.014

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