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GAPS IN THE SATURATION SPECTRUM OF TREES

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Abstract

A graph G is H-saturated if H is not a subgraph of G but the addition of any edge from the complement of G to G results in a copy of H. The minimum number of edges (the size) of an H-saturated graph on n vertices is denoted $\mathbf{sat}(n, H)$, while the maximum size is the well studied extremal number, $\mathbf{ex}(n, H)$. The saturation spectrum for a graph H is the set of sizes of H-saturated graphs between $\mathbf{sat}(n, H)$ and $\mathbf{ex}(n, H)$. In this paper we show that paths, trees with a vertex adjacent to many leaves, and brooms have a gap in the saturation spectrum.

Keywords: saturation spectrum, tree, saturation number.

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