

## ON EDGE COLORINGS OF 1-PLANAR GRAPHS WITHOUT 5-CYCLES WITH TWO CHORDS

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### Abstract

A graph is 1-planar if it can be drawn on the plane so that each edge is crossed by at most one other edge. In this paper, it is proved that every 1-planar graph with maximum degree  $\Delta \geq 8$  is edge-colorable with  $\Delta$  colors if each of its 5-cycles contains at most one chord.

**Keywords:** 1-planar graphs, edge coloring, discharging method.

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