

## THE DISTINGUISHING NUMBER AND DISTINGUISHING INDEX OF THE LEXICOGRAPHIC PRODUCT OF TWO GRAPHS

SAEID ALIKHANI AND SAMANEH SOLTANI

Department of Mathematics  
Yazd University  
89195-741, Yazd, Iran

e-mail: alikhani@yazd.ac.ir  
s.soltani1979@gmail.com

### Abstract

The distinguishing number (index)  $D(G)$  ( $D'(G)$ ) of a graph  $G$  is the least integer  $d$  such that  $G$  has a vertex labeling (edge labeling) with  $d$  labels that is preserved only by the trivial automorphism. The lexicographic product of two graphs  $G$  and  $H$ ,  $G[H]$  can be obtained from  $G$  by substituting a copy  $H_u$  of  $H$  for every vertex  $u$  of  $G$  and then joining all vertices of  $H_u$  with all vertices of  $H_v$  if  $uv \in E(G)$ . In this paper we obtain some sharp bounds for the distinguishing number and the distinguishing index of the lexicographic product of two graphs. As consequences, we prove that if  $G$  is a connected graph with  $\text{Aut}(G[G]) = \text{Aut}(G)[\text{Aut}(G)]$ , then for every natural number  $k$ ,  $D(G) \leq D(G^k) \leq D(G) + k - 1$  and all lexicographic powers of  $G$ ,  $G^k$  ( $k \geq 2$ ) can be distinguished by two edge labels, where  $G^k = G[G[\dots]]$ .

**Keywords:** distinguishing index, distinguishing number, lexicographic.

**2010 Mathematics Subject Classification:** 05C15, 05E18.

### REFERENCES

- [1] M.O. Albertson and K.L. Collins, *Symmetry breaking in graphs*, Electron. J. Combin. **3** (1996) #R18.
- [2] S. Alikhani and S. Soltani, *Distinguishing number and distinguishing index of certain graphs*, Filomat **31** (2017) 4393–4404.  
doi:10.2298/FIL1714393A
- [3] E. Bird, G. Curtis and D.J. Kleitman, *Automorphisms of lexicographic products*, Discrete Math. **11** (1975) 191–198.  
doi:10.1016/0012-365X(75)90036-9

- [4] M. Chan, *The distinguishing number of the direct product and wreath product action*, J. Algebraic Combin. **24** (2006) 331–345.  
doi:10.1007/s10801-006-0006-7
- [5] A. Gorzkowska, R. Kalinowski and M. Pilśniak, *The distinguishing index of the Cartesian product of finite graphs*, Ars Math. Contemp. **12** (2017) 77–87.
- [6] F. Harary, *On the group of the composition of two graphs*, Duke Math. J. **26** (1959) 29–34.  
doi:10.1215/S0012-7094-59-02603-1
- [7] R. Kalinowski and M. Pilśniak, *Distinguishing graphs by edge colourings*, European J. Combin. **45** (2015) 124–131.  
doi:10.1016/j.ejc.2014.11.00
- [8] S. Klažar and X. Zhu, *Cartesian powers of graphs can be distinguished by two labels*, European J. Combin. **28** (2007) 303–310.  
doi:10.1016/j.ejc.2005.07.001
- [9] F. Michael and I. Garth, *Distinguishing colorings of Cartesian products of complete graphs*, Discrete Math. **308** (2008) 2240–2246.  
doi:10.1016/j.disc.2007.04.070
- [10] G. Sabidussi, *The composition of graphs*, Duke Math. J. **26** (1959) 693–696.  
doi:10.1215/S0012-7094-59-02667-5

Received 5 August 2016  
 Revised 28 February 2017  
 Accepted 28 February 2017