

THE DISTINGUISHING NUMBER AND DISTINGUISHING INDEX OF THE LEXICOGRAPHIC PRODUCT OF TWO GRAPHS

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Abstract

The distinguishing number (index) $D(G)$ ($D'(G)$) of a graph G is the least integer d such that G has a vertex labeling (edge labeling) with d labels that is preserved only by the trivial automorphism. The lexicographic product of two graphs G and H , $G[H]$ can be obtained from G by substituting a copy H_u of H for every vertex u of G and then joining all vertices of H_u with all vertices of H_v if $uv \in E(G)$. In this paper we obtain some sharp bounds for the distinguishing number and the distinguishing index of the lexicographic product of two graphs. As consequences, we prove that if G is a connected graph with $\text{Aut}(G[G]) = \text{Aut}(G)[\text{Aut}(G)]$, then for every natural number k , $D(G) \leq D(G^k) \leq D(G) + k - 1$ and all lexicographic powers of G , G^k ($k \geq 2$) can be distinguished by two edge labels, where $G^k = G[G[\dots]]$.

Keywords: distinguishing index, distinguishing number, lexicographic.

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REFERENCES

- [1] M.O. Albertson and K.L. Collins, *Symmetry breaking in graphs*, Electron. J. Combin. **3** (1996) #R18.
- [2] S. Alikhani and S. Soltani, *Distinguishing number and distinguishing index of certain graphs*, Filomat **31** (2017) 4393–4404.
doi:10.2298/FIL1714393A
- [3] E. Bird, G. Curtis and D.J. Kleitman, *Automorphisms of lexicographic products*, Discrete Math. **11** (1975) 191–198.
doi:10.1016/0012-365X(75)90036-9

- [4] M. Chan, *The distinguishing number of the direct product and wreath product action*, J. Algebraic Combin. **24** (2006) 331–345.
doi:10.1007/s10801-006-0006-7
- [5] A. Gorzkowska, R. Kalinowski and M. Pilśniak, *The distinguishing index of the Cartesian product of finite graphs*, Ars Math. Contemp. **12** (2017) 77–87.
- [6] F. Harary, *On the group of the composition of two graphs*, Duke Math. J. **26** (1959) 29–34.
doi:10.1215/S0012-7094-59-02603-1
- [7] R. Kalinowski and M. Pilśniak, *Distinguishing graphs by edge colourings*, European J. Combin. **45** (2015) 124–131.
doi:10.1016/j.ejc.2014.11.00
- [8] S. Klavžar and X. Zhu, *Cartesian powers of graphs can be distinguished by two labels*, European J. Combin. **28** (2007) 303–310.
doi:10.1016/j.ejc.2005.07.001
- [9] F. Michael and I. Garth, *Distinguishing colorings of Cartesian products of complete graphs*, Discrete Math. **308** (2008) 2240–2246.
doi:10.1016/j.disc.2007.04.070
- [10] G. Sabidussi, *The composition of graphs*, Duke Math. J. **26** (1959) 693–696.
doi:10.1215/S0012-7094-59-02667-5

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