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SOME PROGRESS ON THE DOUBLE ROMAN DOMINATION IN GRAPHS

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Abstract

For a graph $G = (V, E)$, a double Roman dominating function (or just DRDF) is a function $f : V \longrightarrow \{0, 1, 2, 3\}$ having the property that if $f(v) = 0$ for a vertex v, then v has at least two neighbors assigned 2 under f or one neighbor assigned 3 under f, and if $f(v) = 1$, then vertex v must have at least one neighbor w with $f(w) \geq 2$. The weight of a DRDF f is the sum $f(V) = \sum_{v \in V} f(v)$, and the minimum weight of a DRDF on G is the double Roman domination number of G, denoted by $\gamma_{dR}(G)$. In this paper, we derive sharp upper and lower bounds on $\gamma_{dR}(G) + \gamma_{dR}(\overline{G})$ and also $\gamma_{dR}(G)\gamma_{dR}(\overline{G})$, where $\overline{G}$ is the complement of graph G. We also show that the decision problem for the double Roman domination number is NP-complete even when restricted to bipartite graphs and chordal graphs.

Keywords: Roman domination, double Roman domination.

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