

SOME PROGRESS ON THE DOUBLE ROMAN DOMINATION IN GRAPHS

NADER JAFARI RAD

Department of Mathematics
Shahed University, Tehran, Iran

e-mail: n.jafarirad@gmail.com

AND

HADI RAHBANI

Department of Mathematics
Shahrood University of Technology
Shahrood, Iran

Abstract

For a graph $G = (V, E)$, a double Roman dominating function (or just DRDF) is a function $f : V \rightarrow \{0, 1, 2, 3\}$ having the property that if $f(v) = 0$ for a vertex v , then v has at least two neighbors assigned 2 under f or one neighbor assigned 3 under f , and if $f(v) = 1$, then vertex v must have at least one neighbor w with $f(w) \geq 2$. The weight of a DRDF f is the sum $f(V) = \sum_{v \in V} f(v)$, and the minimum weight of a DRDF on G is the double Roman domination number of G , denoted by $\gamma_{dR}(G)$. In this paper, we derive sharp upper and lower bounds on $\gamma_{dR}(G) + \gamma_{dR}(\overline{G})$ and also $\gamma_{dR}(G)\gamma_{dR}(\overline{G})$, where $\overline{G}$ is the complement of graph G . We also show that the decision problem for the double Roman domination number is NP-complete even when restricted to bipartite graphs and chordal graphs.

Keywords: Roman domination, double Roman domination.

2010 Mathematics Subject Classification: 05C69.

REFERENCES

- [1] N. Alon and J. Spencer, *The Probabilistic Method* (Wiley-Interscience Series in Discrete Mathematics and Optimization, Wiley, Chichester, 2000).
- [2] M. Aouchiche and P. Hansen, *A survey of Nordhaus-Gaddum type relations*, Discrete Appl. Math. **161** (2013) 466–546.
doi:10.1016/j.dam.2011.12.018

- [3] R.A. Beeler, T.W. Haynes and S.T. Hedetniemi, *Double Roman domination*, Discrete Appl. Math. **211** (2016) 23–29.
doi:10.1016/j.dam.2016.03.017
- [4] S. Bermudo, H. Fernau and J.M. Sigarreta, *The differential and the roman domination number of a graph*, Appl. Anal. Discrete Math. **8** (2014) 155–171.
doi:10.2298/AADM140210003B
- [5] E.W. Chambers, B. Kinnersley, N. Prince and D.B. West, *Extremal problems for Roman domination*, SIAM J. Discrete Math. **23** (2009) 1575–1586.
doi:10.1137/070699688
- [6] M. Chellali and N. Jafari Rad, *Double equality between the roman domination and independent roman domination numbers in trees*, Discuss. Math. Graph Theory **33** (2013) 337–346.
doi:10.7151/dmgt.1669
- [7] E.J. Cockayne, P.A. Dreyer Jr., S.M. Hedetniemi and S.T. Hedetniemi, *Roman domination in graphs*, Discrete Math. **278** (2004) 11–22.
doi:10.1016/j.disc.2003.06.004
- [8] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs* (Marcel Dekker, Inc. New York, 1998).
- [9] M.A. Henning and S.T. Hedetniemi, *Defending the Roman empire—a new strategy*, Discrete Math. **266** (2003) 239–251.
doi:10.1016/S0012-365X(02)00811-7
- [10] Ch.-H. Liu and G.J. Chang, *Roman domination on strongly chordal graphs*, J. Comb. Optim. **26** (2013) 608–619.
doi:10.1007/s10878-012-9482-y
- [11] E.A. Nordhaus and J.W. Gaddum, *On complementary graphs*, Amer. Math. Monthly **63** (1956) 175–177.
doi:10.2307/2306658
- [12] C.S. ReVelle and K.E. Rosing, *Defendens imperium Romanum: a classical problem in military strategy*, Amer. Math. Monthly **107** (2000) 585–594.
doi:10.2307/2589113
- [13] I. Stewart, *Defend the roman empire!*, Sci. Amer. **281** (1999) 136–139.
doi:10.1038/scientificamerican1299-136

Received 31 October 2016

Revised 6 May 2017

Accepted 8 May 2017