

## BOUNDS ON THE SIGNED ROMAN $k$ -DOMINATION NUMBER OF A DIGRAPH

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### Abstract

Let  $k$  be a positive integer. A signed Roman  $k$ -dominating function (SRkDF) on a digraph  $D$  is a function  $f : V(D) \rightarrow \{-1, 1, 2\}$  satisfying the conditions that (i)  $\sum_{x \in N^-[v]} f(x) \geq k$  for each  $v \in V(D)$ , where  $N^-[v]$  is the closed in-neighborhood of  $v$ , and (ii) each vertex  $u$  for which  $f(u) = -1$  has an in-neighbor  $v$  for which  $f(v) = 2$ . The weight of an SRkDF  $f$  is  $\sum_{v \in V(D)} f(v)$ . The signed Roman  $k$ -domination number  $\gamma_{sR}^k(D)$  of a digraph  $D$  is the minimum weight of an SRkDF on  $D$ . We determine the exact values of the signed Roman  $k$ -domination number of some special classes of digraphs and establish some bounds on the signed Roman  $k$ -domination number of general digraphs. In particular, for an oriented tree  $T$  of order

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$n$ , we show that  $\gamma_{sR}^2(T) \geq (n+3)/2$ , and we characterize the oriented trees achieving this lower bound.

**Keywords:** signed Roman  $k$ -dominating function, signed Roman  $k$ -domination number, digraph, oriented tree.

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