

A NOTE ON ROMAN DOMINATION OF DIGRAPHS

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Abstract

A vertex subset S of a digraph D is called a dominating set of D if every vertex not in S is adjacent from at least one vertex in S . The domination number of a digraph D , denoted by $\gamma(D)$, is the minimum cardinality of a dominating set of D . A Roman dominating function (RDF) on a digraph D is a function $f : V(D) \rightarrow \{0, 1, 2\}$ satisfying the condition that every vertex v with $f(v) = 0$ has an in-neighbor u with $f(u) = 2$. The weight of an RDF f is the value $\omega(f) = \sum_{v \in V(D)} f(v)$. The Roman domination number of a digraph D , denoted by $\gamma_R(D)$, is the minimum weight of an RDF on D . In this paper, for any integer k with $2 \leq k \leq \gamma(D)$, we characterize the digraphs D of order $n \geq 4$ with $\delta^-(D) \geq 1$ for which $\gamma_R(D) = \gamma(D) + k$ holds. We also characterize the digraphs D of order $n \geq k$ with $\gamma_R(D) = k$ for any positive integer k . In addition, we present a Nordhaus-Gaddum bound on the Roman domination number of digraphs.

Keywords: Roman domination number, domination number, digraph, Nordhaus-Gaddum.

2010 Mathematics Subject Classification: 05C69, 05C20.

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Received 12 December 2016

Revised 23 March 2017

Accepted 23 March 2017