

ARC FAULT TOLERANCE OF CARTESIAN PRODUCT OF REGULAR DIGRAPHS ON SUPER-RESTRICTED ARC-CONNECTIVITY

GUOZHEN ZHANG AND SHIYING WANG

*School of Mathematical Sciences
Shanxi University
Taiyuan, Shanxi 030006, P.R. China
e-mail: guozhen@sxu.edu.cn*

Abstract

Let $D = (V(D), A(D))$ be a strongly connected digraph. An arc set $S \subseteq A(D)$ is a restricted arc-cut of D if $D - S$ has a non-trivial strong component D_1 such that $D - V(D_1)$ contains an arc. The restricted arc-connectivity $\lambda'(D)$ is the minimum cardinality over all restricted arc-cuts of D . In [C. Balbuena, P. García-Vázquez, A. Hansberg and L.P. Montejano, *On the super-restricted arc-connectivity of s-geodetic digraphs*, Networks **61** (2013) 20–28], Balbuena *et al.* introduced the concept of super- λ' digraphs. In this paper, we first introduce the concept of the arc fault tolerance of a digraph D on the super- λ' property. We define a super- λ' digraph D to be m -super- λ' if $D - S$ is still super- λ' for any $S \subseteq A(D)$ with $|S| \leq m$. The maximum value of such m , denoted by $S_{\lambda'}(D)$, is said to be the arc fault tolerance of D on the super- λ' property. $S_{\lambda'}(D)$ is an index to measure the reliability of networks. Next we provide a necessary and sufficient condition for the Cartesian product of regular digraphs to be super- λ' . Finally, we give the lower and upper bounds on $S_{\lambda'}(D)$ for the Cartesian product D of regular digraphs and give an example to show that the lower and upper bounds are best possible. In particular, the exact value of $S_{\lambda'}(D)$ is obtained in special cases.

Keywords: fault tolerance, restricted arc-connectivity, super-restricted arc-connectivity, Cartesian product, regular digraph.

2010 Mathematics Subject Classification: 05C40, 68M15.

REFERENCES

- [1] D. Bauer, F. Boesch, C. Suffel and R. Tindell, *Combinatorial optimization problems in the analysis and design of probabilistic networks*, Networks **15** (1985) 257–271. doi:10.1002/net.3230150210

- [2] J. Bang-Jensen and G. Gutin, *Digraphs: Theory, Algorithms and Applications* (Springer, London, 2000).
- [3] C. Balbuena, P. García-Vázquez, A. Hansberg and L.P. Montejano, *On the super-restricted arc-connectivity of s -geodetic digraphs*, *Networks* **61** (2013) 20–28.
doi:10.1002/net.21462
- [4] C.W. Cheng and S.Y. Hsieh, *Bounds for the super extra edge connectivity of graphs*, in: *International Computing and Combinatorics Conference*, D. Xu, D. Du, D. Du (Ed(s)), *Lecture Notes in Comput. Sci.* **9198** (2015) 624–631.
doi:10.1007/978-3-319-21398-9_49
- [5] X. Chen, J. Liu and J. Meng, *The restricted arc connectivity of Cartesian product digraphs*, *Inform. Process. Lett.* **109** (2009) 1202–1205.
doi:10.1016/j.ipl.2009.08.005
- [6] A.H. Esfahanian and S.L. Hakimi, *On computing a conditional edge-connectivity of a graph*, *Inform. Process. Lett.* **27** (1988) 195–199.
doi:10.1016/0020-0190(88)90025-7
- [7] Y. Hong, J. Meng and Z. Zhang, *Edge fault tolerance of graphs with respect to super edge connectivity*, *Discrete Appl. Math.* **160** (2012) 579–587.
doi:10.1016/j.dam.2011.10.033
- [8] Z.M. Hong and J.M. Xu, *Vulnerability of super edge-connected networks*, *Theoret. Comput. Sci.* **520** (2014) 75–86.
doi:10.1016/j.tcs.2013.10.021
- [9] J. Liu and J. Meng, *Super-connected and super-arc-connected Cartesian product of digraphs*, *Inform. Process. Lett.* **108** (2008) 90–93.
doi:10.1016/j.ipl.2008.04.006
- [10] J. Liu, J. Meng and Z. Zhang, *Double-super-connected digraphs*, *Discrete Appl. Math.* **158** (2010) 1012–1016.
doi:10.1016/j.dam.2010.02.004
- [11] L. Volkmann, *Restricted arc-connectivity of digraphs*, *Inform. Process. Lett.* **103** (2007) 234–239.
doi:10.1016/j.ipl.2007.04.004
- [12] S. Wang and S. Lin, *λ' -optimal digraphs*, *Inform. Process. Lett.* **108** (2008) 386–389.
doi:10.1016/j.ipl.2008.07.008
- [13] J.M. Xu, *Connectivity of Cartesian product digraphs and fault-tolerant routings of generalized hypercubes*, *Appl. Math. J. Chinese Univ.* **13** (1998) 179–187.
doi:10.1007/s11766-998-0039-x
- [14] J.M. Xu, *Topological Structure and Analysis of Interconnection Networks* (Kluwer Academic Publishers, Dordrecht, 2001).
doi:10.1007/978-1-4757-3387-7
- [15] C. Yang and J.M. Xu, *Reliability of interconnection networks modeled by Cartesian product digraphs*, *Networks* **52** (2008) 202–205.
doi:10.1002/net.20231

- [16] G. Zhang, *Arc fault tolerance of Cartesian product digraphs on hyper arc connectivity*, Int. J. Comput. Math. **91** (2014) 2152–2162.
doi:10.1080/00207160.2014.883069
- [17] Z. Zhang and Y. Zhu, *Cyclic arc-connectivity in a Cartesian product digraph*, Appl. Math. Lett. **23** (2010) 796–800.
doi:10.1016/j.aml.2010.03.013

Received 10 October 2016

Accepted 7 June 2017