

NEW FORMULAE FOR THE DECYCLING NUMBER OF GRAPHS

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Abstract

A set S of vertices of a graph G is called a decycling set if $G - S$ is acyclic. The minimum order of a decycling set is called the decycling number of G , and denoted by $\nabla(G)$. Our results include: (a) For any graph G ,

$$\nabla(G) = n - \max_T \{\alpha(G - E(T))\},$$

where T is taken over all the spanning trees of G and $\alpha(G - E(T))$ is the independence number of the co-tree $G - E(T)$. This formula implies that computing the decycling number of a graph G is equivalent to finding a spanning tree in G such that its co-tree has the largest independence number. Applying the formula, the lower bounds for the decycling number of some (dense) graphs may be obtained. (b) For any decycling set S of a k -regular graph G ,

$$|S| = \frac{1}{k-1}(\beta(G) + m(S)),$$

where $\beta(G) = |E(G)| - |V(G)| + 1$ and $m(S) = c + |E(S)| - 1$, c and $|E(S)|$ are, respectively, the number of components of $G - S$ and the number of

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edges in $G[S]$. Hence S is a ∇ -set if and only if $m(S)$ is minimum, where ∇ -set denotes a decycling set containing exactly $\nabla(G)$ vertices of G . This provides a new way to locate $\nabla(G)$ for k -regular graphs G . (c) 4-regular graphs G with the decycling number $\nabla(G) = \left\lceil \frac{\beta(G)}{3} \right\rceil$ are determined.

Keywords: decycling number, independence number, cycle rank, margin number.

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REFERENCES

- [1] M.O. Albertson and D.M. Berman, *The acyclic chromatic number*, Congr. Numer. **17** (1976) 51–69.
- [2] S. Bau and L.W. Beineke, *The decycling number of graphs*, Australas. J. Combin. **25** (2002) 285–298.
- [3] L.W. Beineke and R.C. Vandell, *Decycling graphs*, J. Graph Theory **25** (1997) 59–77.
doi:10.1002/(SICI)1097-0118(199705)25:1\$(<\$59::AID-JGT4\$>\$3.0.CO;2-H
- [4] R. Diestel, *Graph Theory* (Springer Verlag, New York, 1997).
doi:10.4171/OWR/2007/16
- [5] P. Erdős, *Maximum induced trees in graphs*, J. Combin. Theory Ser. B **41** (1986) 61–79.
doi:10.1016/0095-8956(86)90028-6
- [6] R. Focardi, F.L. Luccio and D. Peleg, *Feedback vertex set in hypercubes*, Inform. Process. Lett. **76** (2000) 1–5.
doi:10.1016/S0020-0190(00)00127-7
- [7] M.L. Furst, J.L. Gross and L.A. Mcgeoch, *Finding a maximum genus graph imbedding*, J. Assoc. Comput. Mach. **35** (1988) 507–537.
- [8] L. Gao, X. Xu, J. Wang, D. Zhu and Y. Yang, *The decycling number of generalized Petersen graphs*, Discrete Appl. Math. **181** (2015) 297–300.
doi:10.1016/j.dam.2014.09.005
- [9] F. Harary, *Graph Theory* (Academic Press, New York, 1967).
- [10] G. Kirchhoff, *Über die Auflösung der Gleichungen, auf welche man bei der Untersuchung der linearen Verteilung galvanischer Ströme geführt wird*, Ann. Phys. Chem. **72** (1847) 497–508.
- [11] R.M. Karp, *Reducibility among combinatorial problems*, in: Complexity of Computer Computations (Plenum Press, New York, 1972) 85–103.
- [12] M.Y. Lien, H.L. Fu and C.H. Shih, *The decycling number of $P_m \times P_n$* , Discrete Math. Algorithms Appl. **3** (2014) 1–9.
doi:10.1142/S1793830914500335

- [13] D.A. Pike and Y. Zou, *Decycling cartesian products of two cycles*, SIAM J. Discrete Math. **19** (2005) 651–663.
doi:10.1137/S089548010444016X
- [14] N. Punnim, *The decycling number of cubic graphs*, Combinatorial Geometry and Graph Theory **3330** (2005) 141–145.
doi:10.1007/978-3-540-30540-8-16
- [15] N. Punnim, *The decycling number of cubic planar graphs*, in: Proceedings of the 7th China-Japan Conference on Discrete Geometry, Combinatorics and Graph Theory, Lecture Notes in Comput. Sci. **4381** (2007) 149–161.
doi:10.1007/978-3-540-70666-3-16
- [16] H. Ren and S. Long, *The decycling number and maximum genus of cubic graphs*, J. Graph Theory **88** (2018) 375–384.
doi:10.1002/jgt.22218
- [17] E. Speckenmeyer, *On feedback vertex sets and nonseparating independent sets in cubic graphs*, J. Graph Theory **12** (1988) 405–412.
doi:10.1002/jgt.3190120311
- [18] E. Wei, Y. Liu and Z. Li, *Decycling number of circular graphs*, ISORA'09 **1** (2009) 387–393.
- [19] E. Wei and Y. Li, *Decycling number and upper-embeddibility of generalized Petersen graphs*, Acta Math. Sinica (Chin. Ser.) **56** (2013) 211–216.
- [20] N.H. Xuong, *How to determine the maximum genus of a graph*, J. Combin. Theory Ser. B **26** (1979) 226–232.
doi:10.1016/0095-8956(79)90058-3

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