

AN EFFICIENT POLYNOMIAL TIME APPROXIMATION SCHEME FOR THE VERTEX COVER P_3 PROBLEM ON PLANAR GRAPHS

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Abstract

Given a graph $G = (V, E)$, the task in the vertex cover P_3 (VCP_3) problem is to find a minimum subset of vertices $F \subseteq V$ such that every path of order 3 in G contains at least one vertex from F . The VCP_3 problem remains NP-hard even in planar graphs and has many applications in real world. In this paper, we give a dynamic-programming algorithm to solve the VCP_3 problem on graphs of bounded branchwidth. Using the dynamic programming algorithm and the Baker's EPTAS framework for NP-hard problems, we present an efficient polynomial time approximation scheme (EPTAS) for the VCP_3 problem on planar graphs.

Keywords: combinatorial optimization, vertex cover P_3 problem, branchwidth, planar graphs, EPTAS.

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