Discussiones Mathematicae
Graph Theory 39 (2019) 55–65
doi:10.7151/dmgt.2060

AN EFFICIENT POLYNOMIAL TIME APPROXIMATION SCHEME FOR THE VERTEX COVER P_3 PROBLEM ON PLANAR GRAPHS

Jianhua Tu

School of Science
Beijing University of Chemical Technology
Beijing 100029, China

e-mail: tujh81@163.com

AND

Yongtang Shi [1]

Center for Combinatorics and LPMC
Nankai University
Tianjin 300071, China

e-mail: shi@nankai.edu.cn

Abstract

Given a graph $G = (V, E)$, the task in the vertex cover P_3 (VCP_3) problem is to find a minimum subset of vertices $F \subseteq V$ such that every path of order 3 in G contains at least one vertex from F. The VCP_3 problem remains NP-hard even in planar graphs and has many applications in real world. In this paper, we give a dynamic-programming algorithm to solve the VCP_3 problem on graphs of bounded branchwidth. Using the dynamic programming algorithm and the Baker's EPTAS framework for NP-hard problems, we present an efficient polynomial time approximation scheme (EPTAS) for the VCP_3 problem on planar graphs.

Keywords: combinatorial optimization, vertex cover P_3 problem, branchwidth, planar graphs, EPTAS.

2010 Mathematics Subject Classification: 05C85.

References

[1] B.S. Baker, *Approximation algorithms for NP-complete problems on planar graphs*, J. ACM **41** (1994) 153–180. doi:10.1145/174644.174650

[1] Corresponding author.

[2] F. Bienstock and C.L. Monma, *On the complexity of embedding planar graphs to minimize centain distance measure*, Algorithmica **5** (1990) 93–109. doi:10.1007/BF01840379

[3] H.L. Bodlaender, M. Cygan, S. Kratsch and J. Nederlof, *Deterministic single exponential time algorithms for connectivity problems parameterized by treewidth*, Inform. and Comput. **243** (2015) 86–111. doi:10.1016/j.ic.2014.12.008

[4] R. Boliac, K. Cameron and V.V. Lozin, *On computing the dissociation number and the induced matching number of bipartite graphs*, Ars Combin. **72** (2004) 241–253.

[5] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (Macmillan/Elsevier, London/New York, 1976).

[6] B. Brešar, F. Kardoš, J. Katrenič and G. Semanišin, *Minimum k-path vertex cover*, Discrete Appl. Math. **159** (2011) 1189–1195. doi:10.1016/j.dam.2011.04.008

[7] M.S. Chang, L.H. Chen, L.J. Hung, Y.Z. Liu, P. Rossmanith and S. Sikdar, *An $O^*(1.4658^n)$-time exact algorithm for the maximum bounded-degree-1 set problem*, in: Proceedings of the 31st Workshop on Combinatorial Mathematics and Computation Theory (2014) 9–18.

[8] M.S. Chang, L.H. Chen, L.J. Hung, P. Rossmanith and P.C. Su, *Fixed-parameter algorithms for vertex cover P_3*, Discrete Optim. **19** (2016) 12–22. doi:10.1016/j.disopt.2015.11.003

[9] E.D. Demaine, F.V. Fomin, M. Hajiaghayi and D.M. Thilikos, *Fixed-parameter algorithms for (k, r)-center in planar graphs and map graphs*, ACM Trans. Algorithms **1** (2005) 33–47.

[10] M.R. Fellows, J. Guo, H. Moser and R. Niedermeier, *A complexity dichotomy for finding disjoint solutions of vertex deletion problems*, ACM Trans. Comput. Theory **2** (2011) #5.

[11] F.V. Fomin and D.M. Thilikos, *New upper bounds on the decomposability of planar graphs*, J. Graph Theory **51** (2006) 53–81. doi:10.1002/jgt.20121

[12] Q. Gu and H. Tamaki, *Optimal branch-decomposition of planar graphs in $O(n^3)$ time*, ACM Trans. Algorithms **4** (2008) #30.

[13] O. Hjortas, Branch decompositions of k-outerplanar graphs (Master's Thesis, University of Bergen, Department of Informatics, 2005).

[14] F. Kardoš, J. Katrenič and I. Schiermeyer, *On computing the minimum 3-path vertex cover and dissociation number of graphs*, Theoret. Comput. Sci. **412** (2011) 7009–7017. doi:10.1016/j.tcs.2011.09.009

[15] J. Katrenič, *A faster FPT algorithm for 3-path vertex cover*, Inform. Process. Lett. **116** (2016) 273–278. doi:10.1016/j.ipl.2015.12.002

[16] R.J. Lipton and R.E. Tarjan, *A separator theorem for planar graphs*, SIAM J. Appl. Math. **36** (1979) 177–189.
doi:10.1137/0136016

[17] H. Moser, R. Niedermeier and M. Sorge, *Exact combinatorial algorithms and experiments for finding maximum k-plexes*, J. Comb. Optim. **24** (2012) 347–373.
doi:10.1007/s10878-011-9391-5

[18] M. Novotný, *Design and analysis of a generalized canvas protocol*, in: Proceedings of WISTP 2010, Lecture Notes in Comput. Sci. **6033** (2010) 106–121.
doi:10.1007/978-3-642-12368-9_8

[19] N. Robertson and P.D. Seymour, *Graph minors, X. obstructions to tree-decomposition*, J. Combin. Theory Ser. B **52** (1991) 153–190.
doi:10.1016/0095-8956(91)90061-N

[20] J.H. Tu, *A fixed-parameter algorithm for the vertex cover P_3 problem*, Inform. Process. Lett. **115** (2015) 96–99.
doi:10.1016/j.ipl.2014.06.018

[21] J.H. Tu and F.M. Yang, *The vertex cover P_3 problem in cubic graphs*, Inform. Process. Lett. **113** (2013) 481–485.
doi:10.1016/j.ipl.2013.04.002

[22] J.H. Tu and W.L. Zhou, *A primal-dual approximation algorithm for the vertex cover P_3 problem*, Theoret. Comput. Sci. **412** (2011) 7044–7048.
doi:10.1016/j.tcs.2011.09.013

[23] J.H. Tu and W.L. Zhou, *A factor 2 approximation algorithm for the vertex cover P_3 problem*, Inform. Process. Lett. **111** (2011) 683–686.
doi:10.1016/j.ipl.2011.04.009

[24] B.Y. Wu, *A measure and conquer approach for the parameterized bounded degree-one vertex deletion*, COCOON (2015) 469–480.
doi:10.1007/978-3-319-21398-9_37

[25] M.Y. Xiao and S.W. Kou, *Exact algorithms for the maximum dissociation set and minimum 3-path vertex cover problems*, Theoret. Comput. Sci. **657** (2017) 86–97.
doi:10.1016/j.tcs.2016.04.043

[26] M.Y. Xiao and H. Nagamochi, *Complexity and kernels for bipartition into degree-bounded induced graphs*, Theoret. Comput. Sci. **659** (2017) 72–82.
doi:10.1016/j.tcs.2016.11.011

[27] M. Yannakakis, *Node-deletion problems on bipartite graphs*, SIAM J. Comput. **10** (1981) 310–327.
doi:10.1137/0210022

Received 10 November 2016
Revised 8 May 2017
Accepted 8 May 2017